

EUCLID'S DATA,

Restored to their

TRUE and GENUINE

ORDER,

Agreeable to *Pappus Alexandrinus's* account of them, in his Preface to the Seventh Book of his Mathematical Collections.

By *RICHARD JACK,*

Teacher of Mathematicks.

In this Edition care hath been taken, to correct the errors in the Enuntiations of several Propositions, and their Demonstrations, contained in the Manuscript Copies, as well as in the other Editions; and such Propositions as were wanting are here restored, and the supernumerary Propositions, contained in the other Copies, are here made Corollaries to the Propositions they respectively belong, as mentioned by Pappus.

L O N D O N,

Printed for A. MILLAR in the Strand.

MDCCLVI.



T O

JAMES DAWKINS,

O F

LAVERSTOKE,

Esquire.

SIR,

FROM your laudable Disposition, to restore the Knowledge of Arts and Sciences to that Perfection they were in, amongst the antient GRECIANS, and which you have manifested by your Princely Generosity in the Publication of the Ruins of PALMYRA and BAL-

A 2

BEC,

D E D I C A T I O N.

BEC, it is, that I have presumed to inscribe to you, this Translation of EUCLID's *Data*, restored to their true and genuine Order, as described by PAPPUS ALEXANDRINUS in his Preface to the seventh Book in his Mathematical Collections. I am, Sir, with the greatest Esteem and Gratitude,



Your most obliged

and most obedient

humble Servant

RICHARD JACK.



T H E
P R E F A C E.

*T*O vindicate the analysis of the *Antients* from the cavils of the modern *Algebraists*, and to state so valuable a performance as the *Data* of Euclid in their true and genuine order, as described by Pappus Alexandrinus in the preface to his seventh book of his mathematical collections, are the principal motives, which induced us, to publish a new edition of the *Data*.

From a manual dexterity in calculating an algebraick equation it is, that the *Moderns* imagine themselves endued with a superior degree of knowledge, and flatter themselves that all mathematical learning depends upon a ready arrangement of their characters, by which, in a few lines, they demonstrate the most

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intricate theorem, and solve the most difficult problem; whereas, on the other hand, the demonstrations of the theorems, and construction of the problems, by the method of the Antients, are extremely tedious and unnecessary. Such is the language of the modern Algebraists. How far they have reason and truth on their side, we appeal to such candid Geometricians who have with care perused the Data, which were designed by their incomparable author as an introduction to the analysis of the Antients, as his Elements were for the principles of Geometry, together with such fragments of the Antients that have escaped the inquiry of time; to such only can we appeal, because it is impossible for any one to be a competent judge without a tolerable share of knowledge in both.

Some Algebraists alledge, the whole life of one man is too small a time, to peruse one half of the modern discoveries, if wrote in the style of the Antients; whereas, as they now stand, the whole may be run over in a few years. But if the question was put, what are those numerous discoveries made by the Moderns? what is that great increase of knowledge which the Algebraists have enriched mathematical learning with, and which the Anti-

ents



ents knew nothing of? I am afraid the answer would turn out, *hic labor hoc opus est*. It is true, the Moderns have made several useful improvements; but, when these improvements are carefully considered, they flow from geometrical, and not from algebraical principles.

It must be acknowledged, that, in this and the last century, a multitude of performances have appeared from the hands of the Algebraists; but, when looked into, most of them, without extending their principles, treat of the same things, digested into systems, formed agreeable to the opinion of each calculator: Others, who claim the honour of extending their principles, treat of what they call Fluxions, calculus differentialis, infiniment Petit's, extreme and ultimate ratios, &c. and with so much obscurity, that no distinct idea of the thing treated, is, communicated to the mind of the reader. From their want of that precision and perspicuity which the Antients carefully observed in all their writings, the mind becomes clouded with confusion; begins to doubt, which terminates in a disbelief of their principles; for which reason they have often been called upon to demonstrate them: But as no demonstration has appeared,

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we may justly conclude, that either their principles were incapable of being demonstrated, or, that the Algebraists are not so well acquainted with demonstration as to be able for the task.

That they perform their solutions in a more easy and expeditious manner, than that practised by the *Antients*, which they so loudly and universally proclaim, is far from being allowed: The most certain, is, always the most expeditious method of enquiring after truth. The proverb *festina lente* is very properly applied to an inquiry of that nature: Of this the most judicious of them are very sensible. If they mean, by their method, they can express in a few lines, what would take up a page or more, by the method of the *Antients*, what then? this general stating of the question proves nothing. To prove what they assert, it ought to have been stated thus; by our method, in a few lines, we can express, with the greatest accuracy and perspicuity, what cannot be expressed by the method of the *antient analysis*, in a whole page.

Certainly, that method must be allowed to be the most clear and distinct, which, to the generality of readers, conveys, in the smallest time, a just and adequate idea of the subject.

If

If therefore a greater time be consumed, perfectly to understand their few lines, than what is necessary to read over and comprehend several pages, nay propositions, treated after the manner of the *Antients*; or if, to render them equally intelligible, they cannot express in two pages, what by the *antient analysis* is done in one, without doubt, it must be concluded, that the *antient analysis*, is much to be preferred, both for distinctness and expedition.

That this is the truth, an experience of many years has convinced us. We have often found, that propositions were well understood, when demonstrated after the manner of the *Antients*, in the fourth part of the time, the same could be conceived, when explained after the manner of the *modern analysis*. The whole *Data* of Euclid, which, in all the copies, consists of 95 Theorems, may, after the first principles, be run over in two weeks; whereas, the same number of propositions, to one in similar circumstances, cannot be equally comprehended in at least double that time. These are truths experienced to all those, who have read the *Data*: Therefore, if the whole life of one man, is too small a time to peruse the *modern discoveries*, when demonstrated agreeable to the method of the *Ancients*,

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Ancients, it must be too small, as they now stand, to understand one fourth part of them. The truth is, all we have left of the Antients, may be understood in the same time, that, to any useful purpose, a few volumes of the moderns can be read. To this we may very properly attribute the present general neglect of mathematicall learning, which is of great importance to society.

*The reader, and scholar stand in the same point of light to the author he reads, and the master who teaches, they both apply, with the same view of being instructed, and the progress they make, is always in proportion to the accurate, distinct, and clearness of expression in either; and when obscurity crouds upon them, they naturally desist from reading and learning. To what other cause then can we attribute the present decay of mathematical knowledge, than the universal opinion of the great difficulty, and the vast length of time that is necessary for the acquiring it, but the modern and unnatural introduction of Algebra, Fluxions, extreme and ultimate ratios, into Geometry? This, I grant, is a just charge against the general practice, that at present prevails, in reading and teaching of Mathematicks; but, when properly taught,
and*

and read, agreeable to the practice of the *Antients*, no study can be more easy, none more pleasant and agreeable; and more real mathematical knowledge, is acquired in half the time that is commonly allotted for classical learning, than in a whole life, conformable to the present reigning practice.

That the method of the *Antients*, is infinitely preferable to the analysis of the *Moderns*, Sir Isaac Newton, that prince of philosophers, was perfectly convinced. For that reason he proposed, that in our inquiries into nature, we should proceed by the analysis and composition of the *Antients*, in order to establish it upon a solid foundation, and put an end to all disputes; for upon no other principles, can we so securely rely. This was his opinion, after he became acquainted with the spirit of the *Antients*. And Mr. Pemberton says, in his preface to his view of Sir Isaac's works; " that he had heard him
" censure himself, for not following them more
" closely than he did, and speak with regret
" of his mistake, at the beginning of his
" mathematical studies, in applying himself
" to the works of Decartes, and other algebraical writers, before he had considered
" the elements of Euclid, with that attention,
" so

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" so excellent a writer deserves: That he
 " highly approved of the laudable attempt of
 " Hugo de Omerique, to restore the antient
 " analysis, &c." Since, the greatest philosopher any age can boast of, speaks with so much regard to the Antients, and in so eminent a manner, prefers them to the Moderns, as to regret his mistake, in reading of algebraical authors, what mathematician, what philosopher can doubt, that the antient analysis is attended with infinitely superior advantages above that of the modern Algebra, extreme and ultimate ratios? &c. Mr. M'claurin, in his account of his Fluxions, Phil. Trans. No. 468. inter alia, says, " unbounded
 " liberties have been introduced of late, by
 " which Geometry is filled with mysteries,
 " and philosophy is likewise perplexed." Again, in the same page, " Geometry has been
 " always considered as our surest bulwark
 " against the subtilties of the sceptics, who
 " are ready to make use of any advantages
 " that may be given them against it, &c." Several other passages to the same purpose are to be found in his account of Sir Isaac Newton's philosophical discoveries. Such is the opinion of one, concerning the method of the Antients, who was a great algebraist himself,

and

and who has wrote two large quartos, in defence of the doctrine of Fluxions. But such is the force of truth, it breaks out in full vigour, in opposition to the strongest prejudices.

If the opinions of real mathematicians can have any weight, many might be produced. The greatest amongst the Moderns, declare in favour of the analysis of the Antients. Sir Isaac Newton's, and Mr. McLaurin's, already have been mentioned; we shall only insert Mr. Powel's, concerning de Omerique's analysis, and that of Mr. Simson's, the greatest geometrician the present age can boast of, in his preface to his Apollonii Pergaei Locorum Planorum, &c. and refer the Reader, for his further satisfaction, to the few treatises the world, as yet, has been favoured with, on the subject, particularly to Simson's Loci, a work the most valuable amongst the modern productions, and not unworthy of Archimedes himself. The length of time to peruse Authors on this valuable subject, can plead no excuse for our neglect, as the abovementioned Author, Halley's Apollonius de Sectione Rationis, de Omerique's analysis, constitute the chief now extant.

Mr.

Mr. Powel, who was professor of Mathematicks in the college of Guada, in giving his opinion of de Omerique's analysis, says,
 “ et ex certa scientia præeligit commo-
 “ diores ut utilius problema simul solvat,
 “ construat & demonstret. Elementa in-
 “ quam rite concinnata adhibet, nam quæ
 “ sexcenti alii interpretes commentati sunt,
 “ ideo formidine cujusque legentis animum
 “ inquietarunt, quod laborarent ignorantia
 “ hujusci finis, five, ut ita dicam, objecti
 “ attributionis unice docentis quæ & qualia
 “ esse debeant elementa ad se conducentia;
 “ unde asserere audeam plus emolumenti
 “ ex hujus analyseos introductione in ele-
 “ menta redundare, quam ex sexcentorum
 “ hocusque interpretum lucubrationibus.
 “ Hanc nudum potius descriptionem, quam
 “ exquisitam laudem expeditit complacen-
 “ tia mea quadriennis, quam in dies augere
 “ non desinit, jugis hujus analyseos exer-
 “ citatio, &c.

Mr. Simson, in his preface to his Apollonius, after he takes notice of particular æquations used by the marquis de L'hospital and Schooten for the solution of different propositions in Apollonius's Loci, gives his opinion concerning the modern algebraical ana-
 I lysis

lysis in the following manner ; “ aliud enim
 “ nihil, praeter haec quae dicta sunt, pro
 “ compositione, hoc est constructione et
 “ demonstratione loci propositi affert Schoo-
 “ tenius, atque sic qui viam hanc sequun-
 “ tur, puerilibus hisce operationibus, sibi
 “ et in geometria tyronibus illudunt. Sed
 “ praeterquam quod aequatio haec, eique
 “ similes *ἀγεωμετρικαί* prorsus sunt, quis
 “ non videt multo difficilius fore inve-
 “ nire ipsas p, q, r geometricè, ut earum
 “ ope aequatio construatur, quam aequa-
 “ tionem ipsam invenire ? Canonem autem
 “ $y = p \pm \frac{q}{r}x$ * nihil ad eas inveniendas
 “ conducere ? idemque censendum est de
 “ Canonibus quos tradunt Locorum ad cir-
 “ culum et Locorum solidorum constru-
 “ ctioni inservientibus. Qui vero tantum
 “ hisce calculis algebraicis assueti sunt, nun-
 “ quam, nisi fortuito, vel Theorematum de-
 “ monstraciones, vel Problematum et Loco-
 “ rum solutiones naturales et elegantes exhi-
 “ bere valebunt. Et quidem plus auxilij

* This is the ultimate conclusion of one of
 the æquations used by Schooten in his *Exercit.*
Mathem. page 210.

“ tum

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“ tum ad Analyfin tum ad compositionem
 “ Geometricam, ex Propositionibus quae in
 “ praedictis Veterum libris Analyticis conti-
 “ nebantur, quam in omnibus Algebrae
 “ praeceptis, jure expectandum est.

It is an unspeakable loss to the learned world, that Sir Isaac Newton did not follow more closely, the resolution and composition of the Antients, in the demonstrations of the propositions in his Principia; for by that means, an end would have been put to all objections, that have since been made to some of his principles, mankind would have been possessed of a just system of philosophy, and, by that plain and just method of reasoning, it would have been perfectly and universally understood; whereas, as it now stands, it is not only obscure, and understood by a very few, though eminent mathematicians; but demonstrated from principles, that have been greatly objected to; nor are mathematicians yet altogether satisfied concerning their truth. No science can be attended with greater disadvantage, than to have its principles clouded with indispellible obscurities, and crowded with difficulties, which only can be overcome by great assiduity, and expence of time: this naturally puts a stop to the progress of learning; and few have spirit
 and

and resolution enough to attempt the execution of it. That this is a truth, time and experience have clearly proved; for, amongst the great multitude of authors that have appeared, since the first publication of the Principia, and by men of great capacity and extensive learning, none have extended these principles, none have gone so far; nor has any one, as yet, explained them in their full extent, in a plain and intelligible manner, so as to be understood by the common run of mathematicians. For Keil, Gregory, Whiston, Gravesand, Mussenbroeck, Clarke, Defagulier, Helsham, Rowning, Ode, Rutherford, &c. and even M'cLaurin himself, have all dwelt upon the outlines, and seem as if they were afraid to enter upon an explanation, of his demonstrations, of the most speculative and intricate part of his principles; nor has any one of them extended the doctrine of motion, which Mr. M'cLaurin, in his account of Sir Isaac Newton's philosophy, calls the key of nature, so far, as the great Galileo in his Moto Locali, where a great number of curious properties of heavy bodies, ascending and descending by gravity, are, in the most beautiful and elegant manner, demonstrated from the pure principles of Geometry. From the whole it is evident, that the

beauty and elegance of the demonstrations of the principles of science are always in proportion to their accuracy, simplicity, and perspicuity.

On the contrary, anxiety, doubt, and disbelief, are the natural consequences, of intricacy and obscurity. The resolution and composition of the Antients, in our opinion, are conformable to the first, and that of the Modern analysis to the last. To all geometricians these are evident truths, however disagreeable to our modern arithmetical mathematicians.

Nor, till such time that the manner of investigating geometrical truths, conformable to the practice of the Antients, is restored, and universally cultivated, can we expect mathematical learning, and of consequence natural philosophy, to flourish.

But as that inestimable performance of Mr. Simpson's Loci, has lately appeared, together with Halley's Apollonius de Sectione Rationis, we hope that these will be soon followed by others, extending these principles: and, since the whole depends upon a previous knowledge of Euclid's Data, and having found, by comparing the different editions, with that account of them given by Pappus Alexandrinus, that several corruptions and inaccuracies have been introduced; we imagined, at present, we could

could not contribute more to the forwarding so valuable a purpose, as to give an edition of the Data, restored free of these errors and corruptions, and to restore such propositions as appeared to be left out, by which, these two valuable performances will be more easily read, and more clearly understood. That our readers may have a more clear and distinct idea of the Data, and of the resolution and composition of the Antients, in general, we shall give them the substance of Pappus's explanation of them, in his Preface to his Seventh Book of his Mathematical Collections, as it stands in Commandine's Edition, and is as follows.

The substance of Pappus's account of the antient analysis.

" To you the son of Hermodorus, I shall
 " here briefly explain the nature of the Loci,
 " which is called resolution. Resolution is
 " the method by which geometricians investi-
 " gate the solution of problems. This they
 " do, by comparing the principles contained
 " in the common elements of Geometry with
 " one another. This method was invented,
 " chiefly for the solution of problems, and
 " which they commonly introduced, immedi-
 " ately

“ ately after the first principles of Geometry.
 “ Upon this subject, three authors have wrote,
 “ viz. Euclid, who is author of the elements;
 “ Apollonius Pergæus; and Aristæus se-
 “ nior; and all proceed, by the method of
 “ resolution and composition.

“ The method of resolution then is, that
 “ which is required, is supposed actually to be
 “ done; and from that supposition, they pro-
 “ ceed, step by step, till they are led to a
 “ certain conclusion, from which the com-
 “ position, or, in other words, the real
 “ construction of the problem itself, may be
 “ performed. In resolution, therefore, that
 “ which is sought, or required to be done, is
 “ taken for granted, or supposed to be found,
 “ or done; then we consider, what that sup-
 “ position corresponds with, or to what it has
 “ any immediate relation; after which, we
 “ again consider, what are the peculiar pro-
 “ perties, of that former corresponding step:
 “ in this manner do we proceed, till at last
 “ we are led, to some property already known,
 “ or some property, contained in the common
 “ principles formerly demonstrated. Such a
 “ process as this, we call resolution, or the
 “ solution reverted, that is, as if the solution
 “ was performed in an inverted manner.

“ Com-

“Composition, on the other hand, is per-
 “formed in a contrary manner; for in it,
 “we take that last concluding step, which we
 “found in the former process for granted:
 “we then consider, what are the consequences
 “that flow from that step; then, in the same
 “manner, as in the preceding process, we
 “proceed, step by step, in a just order, till,
 “at last, we naturally are led to the con-
 “struction of the problem required; and this
 “method is called composition*.

“ Reso-

* To illustrate this, take the following
 problem, which is the ninth proposition in
 the first book of *Euclid*.

To bisect a given right lined angle.

Let BAC be a given right lined angle, it
 is required to bisect it, or to cut it into two
 equal parts.

R E S O L U T I O N.

Suppose it done, and let AF be the line
 that bisects the angle BAC, take any point
 in the line AB, as D, make AE equal to
 AD, and join DE, about the center D, with
 the distance DE, describe the circle EFG,

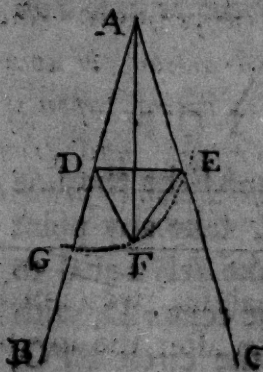
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*“ Resolution is divided into two kinds,
 “ or species; the first kind or species is the
 “ en-*

which cuts AF at F , join the lines DF , FE . Then, because AD is equal to AE , and AF is common, and the contained angle DAF is equal to the contained angle EAF , therefore, by the fourth proposition, the base DF is equal to the base FE ; but DF is equal to DE , being radii of the same circle, wherefore the triangle DEF is equilateral.

C O M P O S I T I O N.

Take any point in the line AB , as D , make AE equal to it, join DE , upon DE describe the equilateral triangle DEF , and join AF ; I say the angle BAC is bisected with the line AF . Because the two triangles ADF , AEF have the side AD equal to AE , and AF common, and the base DF equal to the



*" enquiring, or searching after any truth,
" and is called speculative or theoretical *;
the*

the base EF, being sides of the equilateral triangle, therefore, by the 8th proposition, in the first book, the angle DAF is equal to the angle EAF, subtending the equal sides DF and EF, consequently the angle BAC is bisected with the line AF, which was to be done †.

† Here we first suppose the problem actually done, and then proceed, first by a construction, and then a demonstration, from the antecedent principles in Euclid, until we find the triangle DEF to be equilateral, which finishes the resolution, because it lays open the practical rule for constructing the problem itself, and this, is, what makes the composition. And hence it is, that, in the construction, or composition, after the line DE is joined, upon it, an equilateral triangle is made, and from that step the truth of the construction is proved.

* This part of the analysis, was introduced a considerable time, after the inven-

“ the other is the investigating, or perform-
 “ ing of any thing which is proposed to be
 “ found, or done; and this is called practi-
 “ cal or problematical.

“ In the theoretical or speculative species
 “ of resolution, that property which is the
 “ subject of our inquiry, is taken for granted
 “ to be true, or supposed to exist, the conse-
 “ quence which follows from that supposition,
 “ is also considered as a truth; we then reason
 “ from one consequence, to the next immediate
 “ following one, and in this manner, do we
 “ proceed, step by step, until we arrive at
 “ some simple property: If this last consequen-
 “ tial property be true, the proposition, which
 “ was the subject of our enquiry, is also true;
 “ and the demonstration of it, is performed, by
 “ beginning with the last consequential step of
 “ the resolution, which constituted the known
 “ property: we then proceed, in a reverse
 “ order, through all the steps made use of
 “ in the resolution, which, at last, conduct
 “ us to the truth of the proposition. But
 “ if.

tion of the first part abovementioned, which
 was contrived, as Pappus informs us, solely
 for the solution of problems.

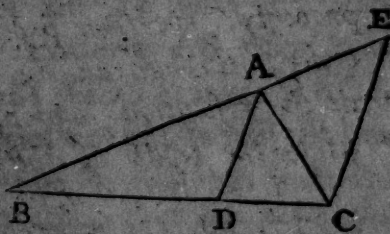
“ if, on the contrary, the last concluding step
 “ in the resolution, which constituted the
 “ known principle, be false, then that, which
 “ was enquired after, is also false*.

“ Again,

* To illustrate this, we shall take the following theorem, which is the first part of the third proposition in the sixth book of *Euclid*.

If any angle in a triangle be bisected, and the line which bisects the angle, cuts the base, then will the segments of the base, have the same proportion to one another, as the other two sides of the triangle have.

Let ABC be a triangle which has the angle BAC bisected with the line AD, and let AD, cut the base BC at D; I say BD, has the same proportion to DC that the side AB, has to the side AC. Let it be true, produce BA, and cut a part of it, as AE, equal to AC, and join EC.



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“ Again, in the problematrical or practical
 “ species of resolution, that which is proposed
 “ to be found or performed is taken for
 “ granted, or supposed to be actually done,
 “ then,

Then, because AE is equal to AC, as BA is to AE, so is BA to AC, (by prop. 7. lib. 5.) but as BA is to AC, so is BD to DC, therefore, (by prop. 11. lib. 5.) as BD is to DC, so is BA to AE, wherefore, (by prop. 2. lib. 6.) AD is parallel to EC*.

* By this demonstration we have proved EC parallel to AD, which is true, because from the point C, a line can be drawn parallel to AD, (by prop. 31. lib. 1.) which line must meet with BA produced: because the angle ADB is equal to ECD, (by prop. 29. lib. 1.) and the sum of the two angles ADB, ABD is less than the sum of two right angles, (by prop. 17. lib. 1.) wherefore the sum of the two angles ABD, ECD is less than the sum of two right angles, and consequently the two lines, BA, CE, produced, must meet, (by Ax. 12. lib. 1.)

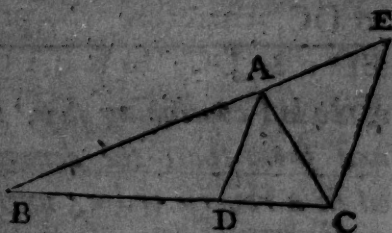
"then, do we consider the consequence that
 "follows from that supposition as true, and
 "in

COMPOSITION.

From the point C draw CE parallel to AD, which cuts AB, produced, at E.

Then, because AD is parallel to EC, and cut with the line BE, the angle BAD is equal to the angle AEC, (by prop. 29. lib. 1.)

For the same reason the angle DAC is equal to



ACE, but the angle BAD is equal to the angle DAC, (by Hypothesis) wherefore the angle ACE is equal to the angle AEC, and consequently (by Prop. 6. lib. 1) the side AC is equal to AE.

Again, because AD is parallel to EC, therefore, (by prop. 2. lib. 6.) as BD is to DC, so is BA to AE; but AE is equal to AC, wherefore as BD is to DC, so is BA to AC*.

* In this, the first step in the construction

“ in this manner do we proceed, considering
 “ the consequences that follow from one ano-
 “ ther, till we are led to some known princi-
 “ ple; if this last concluding step, which con-
 “ stitutes the known principle, can be found,
 “ or done, the proposition proposed, can also
 “ be performed or done. And these, are the
 “ propositions, which mathematicians call
 “ Data *; and the demonstrations of them are
 “ formed

is the last concluding step, in the former part,
 and from that, the demonstration is carried
 on in a reverse order, until we prove the lines
 AC, AE equal to one another, which was the
 first part, in the construction of the resolution,
 and the next step, to that, proves the truth
 of the proposition.

* Marinus, in his preface, defines the word
 datum in this manner: “ The word datum
 “ ought to be defined, by cognitum and por-
 “ tion joined together, or datum is that
 “ which is both known and understood. All
 “ those who define datum after this manner,
 “ give a just and true definition of the word;
 “ neither

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“ formed by beginning with the last conclud-
“ ing step in the resolution, which constituted
“ the known property, and by tracing the
“ steps of the resolution, in an inverted
“ order, we are naturally conducted to the
“ practical construction of the proposition.
“ But,

“ neither can it, with justice or propriety,
“ be defined, by either of these words alone,
“ because things may be known, which are not
“ understood; and, reversely, things may be
“ understood, which are not known; but the
“ word datum comprehends both, that is,
“ any thing, which is said to be given, must be
“ both known and understood, otherwise, it
“ cannot come under the sense, of the word
“ datum or given:” and for that reason, he
joins both in his definition. He also says,
“ those who define it in the following manner,
“ give a very good definition of it, though
“ not so good, nor so general, as that which
“ we have already mentioned, viz. datum
“ or given, is that which we can find, or
“ another equal to it: hence any thing, is
“ said to be given, when we can find ano-
“ ther thing equal to it:” This last, is Euclid’s
own definition.

THE PREFACE

“ But, if the last concluding step, in the
“ resolution, which constituted the known
“ principle, be such, as cannot be performed,
“ or done, in that case, what was proposed
“ to be found or done, is impracticable.

“ Lastly, that which is called *Dierisn*,
“ or Determination, is the manner which
“ explains to us, when, how, in what man-
“ ner, or proportion, or how many ways,
“ a problem may be performed. These are
“ therefore what are called resolution and
“ composition *.

“ The books that have been wrote, con-
“ cerning the abovementioned resolution are
“ ranged

* Hugo de Omerique, in his *Analysis*
Geometrica, begins in this manner †: “ *Af-*
“ *ter*

† “ Postquam elementis Geometricis in-
“ structis fuerit, qui Geometra vocari cupit,
“ fructum colligere laboris sui: aliquam sibi
“ viam paratam habere debet, cujus ope vim,
“ & facultatem cujuscunque cujus eumque
“ Geometricæ propositionis resolutionem in-
“ veniendi acquirat.

“ *Hæc*

The PREFACE.

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"ranged in the following order: First

"Euclid's *Data* in one book: Appollonius's

"*Conic Sections* in two books: *two*

"*Books of Arithmetica* in three books: *three*

"ter the *Elements* of Geometry are under-

"stood, he who has any desire in reaping any

"advantage, by the labour and trouble he has

"gone through, in acquiring the knowledge

"of these principles, ought to have some

"method laid open to him, by the assistance

"of which, and his own genius, he may

"be enabled to acquire the knowledge, and

"discover the demonstration of every geo-

"metrical proposition." This method, he

says, * whatever it is, is called resolution or

analysis, he then proceeds, and, in some mea-

sure, gives the common definition of resolu-

tion,

* "Hæc quidem via, quæcumque illa sit

"analysis, seu resolutio dicitur, & omnium

"consensu ita definitur. Assumptio quæsitæ

"tanquam concessit, per ea, quæ deinceps con-

"sequuntur, ad aliquod concessum procedens.

"Regressus vero a concessio ad quæsitum Syn-

"thesis seu Compositio nominatur. Verum

"enim vero ista definitio nihil nobis certi

"promittit, &c.

“ two books upon proportional Sections; his
 “ two books concerning the Sections of Spaces;
 “ also his two books concerning Tangents:
 “ Euclid’s three books of Porisms: Appol-
 “ lonius’s two books concerning Inclinations
 “ of Lines or Figures; his two books which
 “ treat of the Loci, or places of Plan’s;
 “ his eight books of Conic Sections: Aristæ-
 “ us’s five books, which treat of the Loci
 “ of Solids: Euclid’s two books concerning
 “ the Loci of Surfaces: Eratosthenes’s two
 “ books concerning mean Proportionals.
 “ Hence the whole number of books that
 “ have

tion, which is neither so general, nor so just,
 as that of Pappus’s, which we have already
 mentioned, notwithstanding, he says, it is
 the definition of resolution given by all Au-
 thors; and to this he himself objects, though
 his own definition appears to us much more
 imperfect. He says, “ *Analyſim igitur*
 “ *noſtram nos ita definimus.* Assumptio
 “ *quæſiti tamquam conceſſit, per neceſſa-*
 “ *rias conſequentias ad certum et deter-*
 “ *minatum finem progrediens.* *Hæc equi-*
 “ *dem definitio nobis ſecurum greſſum pro-*
 “ *mittere videtur.*

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" have been wrote upon this subject is thirty.
" one: the contents of which we shall, for
" your perusal, enumerate to the length of
" Appollonius's Conics, together with the
" number of cases of the different propo-
" sitions, the multitude of the propositions,
" loci and diorisms, contained in each book.
" We have also mentioned the number of the
" lemmas which they found necessary to add.
" Nor is there any one proposition, contained
" in these books, but what, to the best of our
" knowledge, we have taken notice of in the
" description of them.

Concerning EUCLID's Data.

" The first book we made mention of, is
" Euclid's Data: which contains ninety
" theorems; the first twenty three propo-
" sitions, treat of the properties of magni-
" tudes in general; the twenty fourth, is,
" concerning proportional lines, without any
" regard to their position. The next four-
" teen following propositions, treat of right
" lines given in position; the next ten are,
" concerning triangles given in species,
" without any regard to their position. The
" next seven propositions, are, concerning
" right lined figures in general, given in

b

Species

" species, without any respect to their position. The next six, treat of the properties of parallelograms, and of spaces given in species, applied to right lines. Of the five following, the first treats of lines, the other four of triangular spaces; that the differences of the squares, described upon the sides of triangles, have, when taken in a certain manner, a given ratio to the areas of the triangles themselves. The next seven propositions, which end at the seventy third, are concerning two parallelograms, which, from the equality of their angles, or having their angles given in magnitude, have a given ratio to one another; and in some copies the same properties of triangles are added as consequences, or corollaries. The next six propositions, that is, from the seventy third to the seventy ninth, the first two of these treat of triangles; the other four are concerning the properties of more than four proportional lines. The three following treat of right lines, which comprehend given spaces*, (which have their sum or

* According to Commandine, in his edition

" difference given, or the difference of their
" squares *.)

" The remaining eight propositions, which
" finish the book, and make it consist of ninety
" propositions, demonstrate properties of cir-
" cles, either given in magnitude only, or
" given in position and magnitude, and con-
" cerning rectangles contained between the
" segments of right lines, cut by the circle,
" and drawn from a given point, which
" rectangles are also given in magnitude.

edition of Pappus, and Doctor Gregory,
in his edition of Euclid's works, it is ex-
pressed thus; " the next three treat of two
" proportional lines which contain given
" spaces." This is plainly corrupted, be-
cause two right lines cannot be said to be
proportional lines; and the second part is
equally absurd. This the Doctor observes
himself, in his explanation of this series.

* That circumstance within the parenthesis
is from Dr. Halley's Edition of the *Sectiones
Rationis*, which explains these three propo-
sitions better than the account given of them
by Commandine or Gregory.

This is the account which Pappus gives of the resolution and composition of the Antients, and of the Data of Euclid. He then proceeds to explain, in the same manner, the contents of the other books, already named, upon this subject: but, as these are foreign to our present purpose, we shall proceed to consider the order of the Data more minutely.

*Dr. Gregory, in his Preface to Euclid's works, says, " that from Pappus's account
 " of the Data, it is plain they have, since
 " his time, been corrupted: for that book
 " did only contain ninety theorems, and now
 " we find it contains ninety five." The
 following order constitutes his divisions of the
 different series. " The first forty eight theo-
 " rems are found in the same order, and treat
 " of the same things which Pappus men-
 " tions; the next seven, to wit, from the
 " forty ninth to the fifty fifth, inclusive,
 " are also as Pappus describes, viz. concern-
 " ing right lined figures in general, given
 " in species, without any regard to their
 " position; these seven Commandine has
 " omitted in his Latin edition; the six fol-
 " lowing, viz. from the fifty sixth to the
 " sixty first, inclusive, likewise correspond
 " to*

“ to Pappus’s account; as also the next
 “ five; for the first of these is concerning
 “ figures described upon two right lines
 “ which have a given ratio to one another;
 “ but in place of the other four there are
 “ five; from whence it appears, without
 “ all manner of doubt, that those which
 “ are commonly reckoned the sixty fourth
 “ and sixty fifth are no more than the two
 “ different cases of one and the same propo-
 “ sition: for the one is the same property of
 “ an acute angled triangle, that the other
 “ is of an obtuse angled one: and hence, the
 “ propositions are augmented one more in
 “ number, than what they were in Pappus’s
 “ time. The next seven differ no other-
 “ wise from Pappus’s description, except
 “ that the sixth or penultimate proposition,
 “ which demonstrates a property, of any right
 “ lined figure whatsoever, and the last which
 “ treats of a right angled parallelogram,
 “ compared with another right lined figure.
 “ Moreover, those which are commonly rec-
 “ koned the seventy first and seventy second,
 “ and in other editions, those which are called
 “ the seventy fifth, and seventy sixth, ought,
 “ by Pappus’s account, to be corollaries to
 “ the last series of propositions: because, in
 “ these,

“ these, the same things are enun-
 “ tiated, and proved of triangles, which the former de-
 “ monstrated of parallelograms. Hence it
 “ is manifest, that here are four propositions
 “ added to the number which Pappus men-
 “ tions. These therefore, together with
 “ the one we have already taken notice of,
 “ make the whole number ninety five. From
 “ thence it follows, that the last proposition
 “ in the last mentioned series, which consti-
 “ tutes, (according to Pappus,) the seventy
 “ third, is now the seventy eighth. A-
 “ mongst the following theorems, (as Pap-
 “ pus says,) two are concerning triangles;
 “ and four that treat of more than four pro-
 “ portional lines; whereas there are only
 “ three of the last mentioned propositions to
 “ be found in all the other editions. Pap-
 “ pus, after this, mentions three theorems,
 “ concerning two proportional right lines,
 “ which comprehend a given space; but
 “ here are four found: In this, there is
 “ unquestionably an error; for two right
 “ lines cannot be said to be proportional
 “ lines: and moreover, either the sum, or
 “ difference of these lines, or of the squares
 “ described upon them, are given. The last
 “ eight propositions exactly correspond to
 “ Pappus’s

" Pappus's description. And hence we have
 " ninety theorems, if we make those which
 " are commonly reckoned the sixty fourth
 " and sixty fifth one proposition, and the
 " 71st, 72d, 75th, and 76th, corollaries
 " to the preceding series, as Pappus inti-
 " mates, and take one from the four in the
 " antepenultimate series, and add one to the
 " three contained in the penultimate series,
 " in that case the Data would be reformed
 " to Pappus's account. We have, however,
 " ranged the propositions in the same order,
 " in which they were in our manuscript;
 " for by altering that order, the citations
 " of them from authors would be thrown
 " into confusion."

This is the Doctor's account of the Data,
 by which he endeavours to point out the
 errors introduced into the manuscripts, and
 to account for, and state in their proper
 order, the ninety theorems mentioned by
 Pappus. The Doctor informs us, he has
 corrected the errors in the Greek text in a
 great many places, and also those committed
 by Claudius Hardy in his Latin edition,
 translated from Bernardo's Greek copy;
 and that he has retained the figures which
 were in the manuscript, and which Hardy

had altered. The 86th theorem, he says, is very remarkably vitiated, nor from the manuscripts can he restore it. He at last has recourse to the modern analysis, and endeavours to give a general solution of it, by a biquadratick equation. This is the substance of what the Doctor observes concerning the Data. We cannot, however, agree with him in his manner of making out Pappus's ninety theorems. The length of the sixty first proposition, which includes the first five series's, without doubt correspond to Pappus's account, and we only differ from him in the enuntiations of the 57th, 58th, and 59th propositions. Because in that series, (which consists of six propositions,) Pappus expressly says, they relate to parallelograms: and though he mentions spaces applied to right lines, yet, from the whole, when the nature of the construction and demonstration of each proposition is considered, and when these are compared to Pappus's account, it evidently follows, that the spaces there expressed, are limited to parallelograms or triangles, otherwise neither the constructions nor demonstrations, there used, could prove the propositions. That the spaces applied to these right lines cannot be interpreted right lined figures in
 general

general is evident; because the demonstrations of these three, are by no means similar to those used in the preceding series, which are applicable to all right lined figures whatsoever; for the first step in each of these demonstrations is limited to parallelograms or triangles, and therefore, if these propositions extended to right lined figures in general, the demonstrations there used are imperfect, and the propositions are not proved, as we have observed in our note to the fifty ninth proposition. For this reason, we have altered the enuntiations, in these three propositions in this series. The Doctor says the next five propositions also agree with Pappus's account, only in place of the other four we find five: this error he corrects, by supposing the 64th and 65th were originally one, because, when considered, they are only different cases of the same proposition; and therefore, by joining them together he accounts for one of the supernumerary propositions. That there is an error in the manuscripts, concerning this series, is an evident truth; but the Doctor, in our opinion, has not completely corrected it. Dr. Halley, in his account, seems to be nearer the truth, when he says, one of these is already named, to wit, the 49th; but, by Dr. Hal-

respond

ley's account, in place of five we have only four: therefore this correction does not correspond to Pappus's account. Our opinion is, that, that which Dr. Halley says is already named, is the 63d proposition, in the other editions, and, as the Doctor observes, is, without doubt the same with the 49th, or, at least, a corollary to it: therefore, if this be taken away, and placed as a corollary to the 49th, which we have done, the series, in that case, exactly corresponds to Pappus's narration. That this error is by no means corrected by joining the 64th and 65th in one, as Dr. Gregory mentions, is evident. For by that manner of correcting it we should have the number, but not the propositions. Because Pappus expressly says, the first is concerning figures described upon right lines, which have a given ratio to one another; the other four are, concerning the difference of the squares described upon the different sides of a triangle, compared to the triangle itself. And in that 63d proposition, the difference of the squares described upon the triangle is not compared to the triangle, but the squares themselves are compared to it, which is no more than the 49th, viz. figures given in species, and described upon the same line,

line, have a given ratio to one another: and hence it is manifest, by Dr. Gregory's correction, that we have the number, but not the propositions themselves. On the contrary, if that proposition be taken away, the remaining four are all concerning the difference of the squares described upon the sides of a triangle, compared to the triangle itself, which perfectly agrees with Pappus's account.

The next series, which, according to Pappus, consists of seven propositions, and eleven in the manuscripts: This series, is much more corrupted than any one, of the whole Data, except the enuntiations in the propositions in the penultimate series. Dr. Gregory says the error may be corrected in this series, by making these four propositions, which treat of triangles, corollaries to the others, which demonstrate the same properties of parallelograms, and which Pappus mentions were made corollaries to these propositions in this series. Though that in some measure, may account for a part of the error, it by no means can correct the whole: The Doctor himself observes, that the penultimate proposition is concerning right lined figures in general: This proposition therefore cannot be one of the seven mentioned by Pappus; and indeed, when this
propa-

proposition is considered, it is the 54th repeated, with a different demonstration: This proposition therefore, is plainly spurious, and ought to be cancelled, which we have done: by so doing, the series falls short of Pappus's number by one; this we have supplied; and that it is the proposition Pappus means, appears from this circumstance, that some add, as corollaries, the same properties of triangles; but what they make the 72d proposition, cannot be a natural corollary to any of these properties of parallelograms, which they have there inserted: Hence that proposition, which proves the same property of parallelograms, which the 72d does of triangles, is plainly the proposition wanting: This therefore, must be the 70th proposition according to Pappus's order. Though we have thus far corrected the error in this series, by leaving out their 77th, and inserting, in its place, this which was wanting we have not completely corrected the error introduced into this series; because, in this case, all the similar properties of triangles, which are demonstrated of parallelograms, are not to be found among the other propositions: These we have likewise supplied, as corollaries to each proposition, to which they respectively belong. Again, in this series Euclid demonstrates, the

converse

converse of each in their regular order: hence the converse of that which we have inserted is wanting; but, if supplied, it will make the series consist of eight and not of seven, agreeable to Pappus: to remedy that, we have made the converse of our proposition a corollary to it, by which means it has two corollaries where the others have only one. It is true, the Doctor observes, that, the last proposition in this series, is particularly limited to a rectangle, which is compared to any right lined figure, from which it is highly probable, that this last proposition is spurious, and added by some other author since Pappus's time; and that which constitutes the converse of ours, and which we have made its first corollary, ought to be the 71st in Pappus's order. This, however, we cannot be so certain of, for which reason we chuse to retain that last one, as it stands in the other copies. Thus far, then, have we corrected the Data to the 73^d proposition, and reduced them to the true order, in which they were laid down by Euclid himself, at least, agreeable to that order, in which they are described by Pappus.

The next series, which consists of six, viz. two of triangles, and four of more than four proportional lines. This series, in all the copies,

copies, is corrupted two ways; for in place of the first two, which relate to triangles, we have three; the error, however, is obvious. Because that which constitutes their 79th is an elementary proposition, and has no manner of connection with the Data, and consequently is spurious; this evidently appears from its enunciation. Indeed, if we allow the first demonstration of the proposition which follows it, to be Euclid's own, in that case, as Euclid has not demonstrated that property, in his Elements, he must have introduced it here, by way of a lemma; which we have done. But, as the second demonstration of this proposition is, by far, more just and elegant, (as we have observed in our note to the lemma) it is consequently more of the Euclidean stile, from which we conclude, that both this lemma, and the first demonstration of that proposition, have been added since Pappus's time. As Dr. Gregory takes no notice of this error, we cannot conceive how he reconciled the first part of this series, with the account given of it by Pappus; for if he allowed this lemma to be one of the propositions, in that case there will be three in the series, which treat of triangles; whereas Pappus mentions only two: on the other hand, if he supposed the preceding proposition

position belonged to the former series, and is one of the number, which he says ought to be made a corollary to one of the propositions in the former series, this, by no means can answer; because that proposition is concerning a triangle given in species, and consequently, is no natural consequence to any of them; and therefore cannot belong to the former series, and consequently, must be the first of the two in this series; hence, by making it the first, and the other a lemma to the 75th, we complete the first part of this series. And, as there is one wanting in the other part, which treats of more than four proportional lines, by considering the nature of these propositions, and the order which Euclid observes in his proportionals, both in his 6th and 7th books, we easily discovered, both the proposition, and the place it ought to be ranged in: for as the first is, If there be three right lines in one order, in continued proportion, and three lines in another order, likewise, in continued proportion, and if the ratio of the extremes in each order be given, the means will also have a given ratio to one another; and, if the ratio of one extreme in one order, to one extreme in the other order be given, and also the ratio of the means to

one another be given, the other extremes will have to one another a given ratio. *The proposition, which, according to Euclid's common method, ought to follow this, is evidently the one we have supplied, viz.* If there be four proportional lines in one order, and four in another order, and if the extremes in each order have a given ratio to another, each to each, and one of the means in each order have also a given ratio to one another; the other two means will likewise have a given ratio to one another; and if the means in each order have a given ratio to one another, each to each, and one of the extremes in each order have a given ratio to one another, the other two extremes will also have a given ratio to one another. *This, I say, is the proposition which naturally ought to follow the abovementioned one, and is, undoubtedly, the fourth which Pappus mentions, and is wanting in all the copies.*

The next series, which Pappus says consists of three propositions, that treat of two lines containing a given space; but in all the copies four are found. This series is likewise corrupted two ways; for, in the first place, they have one more than there really was; and, in the second, they are all falsely enuntiated, in so far

far as, they all mention the word space in general, without particularly specifying what kind of figures are meant by those spaces. And as two lines can never properly be said to comprehend a space, unless it be a four-sided figure, and even that four-sided figure must be a parallelogram, otherwise it would be quite undetermined in its magnitude. This is enuntiating a proposition in a more ambiguous and loose manner, than any geometrician ever did; much less Euclid, who is so conspicuous for his distinctness and precision. We have therefore limited the spaces in the enuntiations of these propositions to parallelograms or triangles, because the same properties take place in triangular spaces that belong to parallelograms (by prop. 15. Euc. lib. 5.) this therefore was certainly Euclid's meaning, as well as Pappus's. And by applying them to any other figure, all the demonstrations are imperfect, and inconclusive (as we have observed in our note to this series). The demonstrations, used in all the copies, not only restrict the figures to parallelograms, but to right angled parallelograms, which plainly proves our assertion, and that the enuntiations in the manuscripts, are very much corrupted. This Dr. Gregory observes; but, as we have already mentioned, he has recourse to a biquadratick equation, to correct the two last; whereas, if the Doctor had corrected the enuntiations, and confined them to parallelograms,

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grams, or triangles, the correction would have been much easier, and more agreeable to the nature of the Data, as well as Euclid's method of treating geometrical subjects. Now, though it is evident, that one of the two last, is spurious, and has been introduced into the Data since Pappus's time, it is, however, difficult to determine which of the two it is. But as that which in the manuscripts is made the 4th, or their 87th proposition, is more similar to the other two, than the third, or their 86th proposition, we have therefore used it as the genuine proposition of Euclid. Dr. Halley appears to be of the same opinion: For, in his Appollonius de Sectione Rationis, when he gives an account of these three propositions from Pappus; to Pappus's account, he adds his own opinion in a parenthesis, which is ("quarum summa vel differentia statatur, vel etiam differentia potestatum.") This differentia potestatum, cannot, in the smallest degree, relate to their 3d or their 86th proposition, and therefore must be the last, or their 87th, or our 82d, which he here describes: But, as this 3d, or their 86th, is a real Data, we have likewise inserted it, without any number annexed to it, that the numbers, according to Pappus's account, may not be interrupted. Hence, by these corrections, which we have above observed, and the last series in

the manuscript copies, exactly corresponding to Pappus's account, we have thereby ranged all the propositions in Euclid's Data, into the order they originally were, agreeable to the description of them by Pappus, and, as we have corrected the errors, introduced as well into the enuntiations as into the demonstrations, since Pappus's time, we hope the candid Reader will in some measure allow it merits the title of Euclid's Data restored.

Dr. Gregory's design in this, as well as in all the other parts of Euclid's works, that are extant, was, to give a Greek and Latin edition in the literal sense, according as they were found in the antient Greek manuscripts, in the libraries of the university of Oxford. Therefore, it was by no means his business to invert the order of the propositions, in the smallest degree, nor yet to alter the expressions in their enuntiations, nor the nature of their demonstrations; neither, in this case, was it his province to enter into a minute discussion of these errors, but only to give a general hint how some of them may be corrected which he has done. As this was the nature of his design, he could not properly proceed otherwise, though he was very sensible of the many errors and corruptions, that have been introduced into the Data, since Pappus's time. In this he succeeded, to the universal satisfaction

of

of the learned world. Our design, as we have already observed, was of a very different nature, it was with that view to correct all these errors, mentioned by the Doctor, and to restore their order, and to conform them to that, which Pappus describes; that by so doing, we might have so valuable a work, which is the key of the analysis of the Antients, in the same form in which Euclid himself wrote them, enuntiated with the same precision, and demonstrated with the same simplicity. How far we have succeeded, we leave to the determination of the candid Reader.

The Reader is desired to correct the following Errors.

PREFACE, Page ix. line 23. for double read quadruple. P. xxxi. l. 20. for concessit r. concessi. P. xxxii. l. 3. r. concessi. DATA, Page 10. line 9. after C, read be given. P. 12. l. 13. for the same, r. some. P. 19. l. 2. for BF r. CF. P. 30. l. 14. after wherefore t. the ratio. P. 52. l. 5. for L r. N. P. 60. l. 19. for FG, r. F. P. 71. l. 7. for square r. squares. P. 72. second Citation, for Euc. lib. 1. r. lib. 6. P. 82. line 24. dele if. P. 87. l. 15. for BC r. BD. P. 88. second Citation, for Dat. 8. r. Dat. 3. P. 95. Citation at the foot of the page for Prop. 35. r. Prop. 1. Euc. lib. 6. P. 97. Citation first for Prop. 19. r. Prop. 18. Ditto, line 29. for have, read has. P. 114. dele lines 27, 28, and 29; and in line 30, dele twice. P. 117. l. 10. for EF r. EH. P. 119. l. 15. for remaining FG r. remaining fide FG. Ditto, last Citation, for Dat. 1. r. Dat. 4. P. 120. line 7. for EG r. FG, and in l. 9. r. have their angles given, and have a given ratio, &c. P. 121. l. 2. for AC r. AG. P. 123. l. 22. dele to Proposition. P. 125. Citation 5. for Prop. 28. r. Prop. 23. P. 126. l. 27. dele to Proposition 70. P. 144. l. 20. for BCA r. BAC. P. 163. l. 2. for AE r. AB. P. 173. l. 13. for DC, read DF.





EUCLID's DATA.

DEFINITIONS.

1st.  *INES*, angles, planes or surfaces, are said to be given in magnitude, when we can find them; or others equal to them.

2d. A ratio, or proportion, is said to be given, when we can find it; or another equal to it.

3d. Right lined figures are said to be given in species, when all their angles are given in magnitude, and the ratios of all their sides are given, each to each.

4th. Points, lines, and angles, are said to be given in position, when they always retain the same situation.

5th. A circle is said to be given in magnitude, when its radius is given in magnitude.

6th. A circle is said to be given in position and magnitude, when its center is given in position, and its radius is given in magnitude.

7th. Segments of circles are said to be given in magnitude, when the angles contained in them,

B

and

EUCLID'S DATA.

and also the right lines that form their bases, are given in magnitude.

8th. Segments of circles are said to be given in position and magnitude, when the angles contained in them are given in magnitude, and the lines which form their bases are given both in position and magnitude.

9th. A magnitude is said to be greater than a magnitude, by a given magnitude, when from it, the given magnitude is taken away, the remainder is equal to that other magnitude. That is, let

A C B AB and D be two unequal magnitudes, whereof AB is the greater, and let E be a given magnitude, and let a part, as BC, be cut off the greater AB, equal to the given magnitude E; then if the remainder AC be equal to the other magnitude D, the greater AB exceeds the smaller D by the given magnitude E.

10th. One magnitude is said to be smaller than another magnitude, by a given magnitude, when the given magnitude, added to the smaller, makes the sum equal to the other magnitude; that is, if

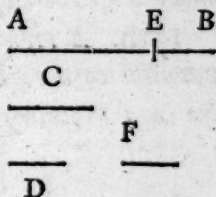
A B E AB and C be two unequal magnitudes, and AB the smaller, and D a given magnitude, and if BE, equal to D, be added to the smaller AB, so that the sum AE is equal to the other magnitude C, then AB is said to be smaller than C by the given magnitude D.

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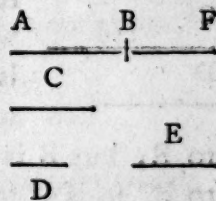
11th.



11th. One magnitude is said to be greater than in a given proportion to another magnitude, by a given magnitude; when from the greater magnitude, the given magnitude being taken away, the proportion of the remainder to the other magnitude, is equal to the given proportion, or ratio: for instance, let AB and C be two magnitudes, F a given magnitude, and D a given ratio; if from AB a part be cut off, as EB, equal to the given magnitude F, and if the ratio of the remainder AE, to the other magnitude C, be equal to the given ratio D; then AB is said to be more than in a given proportion to C, by the given magnitude F.



12th. One magnitude is said to be smaller than in a given proportion to another magnitude, by a given magnitude; when, by adding that given magnitude to it, the sum bears a proportion to the other magnitude equal to the given proportion. For instance, let AB and C be two magnitudes, D a given ratio, and E a given magnitude, and let BF, equal to E, be added to AB; if the ratio of the whole AF to C be equal to the given ratio D; then AB is said to be smaller than in a given proportion to C, by the given magnitude E.



EUCLID'S DATA.

13th. *A right line is said to be drawn down from a given point to a line given in position, when it is drawn, so as to make a given angle with it.*

14th. *A right line is said to be drawn up from a given point to a right line, which is given in position, when it is drawn in such a manner as to make a given angle with it.*

15th. *A right line drawn from a given point, parallel to a right line given in position, is said to lye in a contrary position with that line.*

PROPOSITION I.

If any two magnitudes of the same kind be given, their ratio to one another is also given.

Let A and B be two given magnitudes. I say the ratio of A to B is given. For since A

^a Def. 1. of this.	A B C D 	is given, a magnitude equal to it can be found ^a , which let be C; and because B is given, a magni- tude equal to it can be found, which let be D. Now because A is equal to C, and B equal to D, wherefore ^b as A is to B, so is C to B; but B is equal to D; wherefore ^c as A is to B, so is C to D; but the ratio of C to D is found, and that is equal to the ratio of A to B.
^b Prop. 7. Euc. lib. 5.		Therefore the ratio of A to B is given ^c . There- fore if any two magnitudes of the same kind be given,

^c Def. 2.
of this.

given, their ratio to one another is also given, which was to be demonstrated.

PROPOSITION II.

If a given magnitude has a given proportion to any other magnitude, that other magnitude is also given.

Let A be any given magnitude, and let A have a given proportion to any other magnitude as B. I say B is also given.

Because A is given, therefore one

equal to it may be found^d, which

let be C. And because the ratio

of A to B is given, wherefore a

ratio may be found equal to it^e,

which let be that of C to D.

A

B

C

D

^d Def. 1.
of this.

^e Def. 2.
of this.

Then because A has the same proportion to B that C has to D, and A is equal to C, therefore

B is equal to D^f; hence the magnitude D is

found equal to B^g; therefore B is given.

^f Prop. 14.
Euc. lib. 5.

^g Def. 1.
of this.

Therefore, if a given magnitude has a given proportion to another magnitude, that other magnitude is also given; which was to be demonstrated.

PROPOSITION III.

If there be any number of magnitudes given, that magnitude which is compounded of them all, is also given.

B 3

Let

Let AB and BC be two given magnitudes, I say the magnitude AC, which is compounded of them, is also given. For since AB is given, a magnitude may be found equal to it^h, which let be DE; and because BC is given, a magnitude can be found equal to it, which let be EF. Then because AB is equal to DE, and BC to EF, the whole DF is equal to the whole ACⁱ; hence there is a magnitude DF found equal to AC, wherefore^k AC is given.

^h Def. 1.
of this.

ⁱ Ax. 2.
Euc lib. 1.

^k Def. 1.
of this.

Therefore if there be any number of magnitudes given, that magnitude which is compounded of them all, is also given; which was to be demonstrated.

PROPOSITION IV.

If from a given magnitude, a given magnitude be taken away, the remaining magnitude is also given.

Let the given magnitude AC be taken away from the given magnitude AB. I say the remaining magnitude CB is given. For since AB is given, a magnitude equal to it may be found^l, which let be DF; and since AC is given, let DE be found equal to it. Then because AB is equal to DF, and AC to DE, the remainder CB is equal to the remainder EF^m:
then

^l Def. 1.
of this.

^m Ax. 3.
Euc. lib. 1.

EUCLID'S DATA.

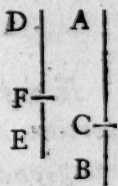
then since EF is found equal to CB, therefore CB is givenⁿ.

Wherefore, if from a given magnitude, &c. of this.ⁿ Def. 1.
which was to be demonstrated.

PROPOSITION V.

If any whole magnitude has a given proportion to its part, the same magnitude will have a given proportion to the remaining part.

Let AB be any magnitude, and AC a part of it, and let AB have a given proportion to AC. I say AB will also have a given proportion to the remainder CB. For let DE be any given magnitude; and because the ratio of AB to AC is given, a ratio equal to it can be found^o, which let be that of DE to DF. Then because the ratio of DE to DF is given, and DE is given, wherefore DF is also given^p. And since the whole DE is given, and the part DF given, therefore the remainder FE^q is given, and consequently the ratio of DE to FE^r is given. Now because AB is to AC, as DE is to DF, by conversion it will be^s, as AB is to CB, so is DE to FE; but the ratio of DE to FE is given; therefore the ratio of AB to CB is given.



^o Def. 2.
of this.

^p Dat. 2.

^q Dat. 4.

^r Dat. 1.

^s Cor.

prop. 19.

Euc. lib. 5.

Therefore if any whole magnitude has a given proportion to one of its parts, it will have a given

E U C L I D's D A T A.

portion to the remaining part; which was to be demonstrated.

COROL. From this proportion it follows, that if the proportion of any whole magnitude to one of its parts be given, the proportion of the parts to one another is given. For since
[†]Prop. 17. AB is to CB as DE is to EF, by division [†] it
 Euc. lib. 5. will be, as AC is to CB, so is DF to FE; but the magnitudes DF and FE are given; where-
^v Dat. 1. fore the ratio of DF to FE is given^v, consequently the ratio of AC to CB is given.

P R O P O S I T I O N VI.

If any number of magnitudes have a given proportion to one another, these magnitudes, when compounded, will have a given proportion to each particular part.

Let the magnitudes AC, CB have a given proportion to one another. I say the ratio of the whole AB to either of the magnitudes AC, or CB is given. For let DE be any given magnitude, and because the ratio of AC to CB is given, wherefore ^v a ratio may be found equal to it, and let that be as DE to EF. Then because AC is to CB as DE is to EF, and the ratio of AC to CB is given, therefore the ratio DE to EF is given; but DE is given in magnitude, therefore EF is given

^v Def. 3.
of this.



given in magnitude^x, and consequently DF^y is ^z Dat. 2.
 given in magnitude. Since DF and FE are ^y Dat. 3.
 both given in magnitude, the ratio of DF to
 FE is given^z. But because AC is to CB, as ^z Dat. 1.
 DE is to EF; by composition^a, as AB is to ^a Prop. 18.
 CB, so is DF to EF; but the ratio of DF to ^{Euc. lib 5.}
 EF is given, wherefore the ratio of AB to CB
 is given. In the same manner it may be de-
 monstrated, that the ratio of AB to AC is
 given.

*Therefore, if there be any number of magnitudes
 which have a given proportion to one another, the
 sum of these magnitudes has a given proportion to
 each particular part; which was to be demon-
 strated.*

PROPOSITION VII.

*If any given magnitude be divided into two parts
 which have a given ratio to one another, each of
 these parts are given in magnitude.*

Let AB be a given magnitude, divided into
 the parts AC, CB, which have a given propor-
 tion to one another. I say each ^{A C B}
 of the parts AC, CB, are given ———|——
 in magnitude, because the ratio of AC to CB
 is given, therefore the ratio of the whole AB
 to AC is given^b; but AB is given, therefore ^b Dat. 6.
 AC is given^c, and consequently^d CB is given. ^c Dat. 2.
^d Dat. 4.

*Therefore, if any given magnitude be divided into
 two parts which have a given ratio to one another,
 each*

each of these parts are given in magnitude; which was to be demonstrated.

PROPOSITION VIII.

Magnitudes which have a given proportion to one and the same magnitude, have a given proportion to one another.

Let A, B, and C, be three magnitudes, and let the ratio of A to B, as also the ratio of B to C; I say the ratio of A to C is given. For let D be a given magnitude. Then because the ratio of A to B is given, a ratio can be found

- ^e Def. 2. equal to it^e, which let be that of D to E; of this. therefore the ratio of D to E is given; but D is given in magnitude, wherefore E is also given in magnitude^f. Again, because the ratio of B to C is given, a ratio can be found equal to it, which let be that of E to F, consequently the ratio of E to F is given; but E is given in magnitude, ^g therefore F is given in magnitude. Then because D and F are two given magnitudes, their proportion is given^h. But because A is to B, as D is to E, and B is to C, as E is to F; therefore by equality, as A is to C, so is D to Fⁱ; but the ratio of D to F is given, ^j wherefore the ratio of A to C is given.

Therefore, magnitudes which have a given proportion to one and the same magnitude, have a given proportion to one another.

portion to one another; which was to be demonstrated.

PROPOSITION IX.

If two, or any number of magnitudes, have a given proportion to one another, and if these magnitudes have a given ratio to a like number of other magnitudes, each to each, though not the same proportions with the former, all these other magnitudes will have a given ratio to one another.

Let A, B, C, be magnitudes, whose ratios to one another are given, and let D, E, and F, be other magnitudes, whose ratios to the first magnitudes

	<u>A</u>	<u>D</u>
A, B, C, are also given. I	B	E
say the ratios of D, E, and F,	<u> </u>	<u> </u>
to one another are all given.	C	F

Because the ratio of A to B is

givenⁱ, and the ratio of A to D is given, therefore the ratio of B to D is given^k. But the ratio of B to E is given^l, therefore the ratio of D to E is given. Again, because the ratio of B to C is given, and the ratio of B to E is given^m, thereforeⁿ the ratio of C to E is given; but the ratio of C to F is given; therefore the ratio of E to F is given^o. Also since the ratio of D to E is given, and the ratio of E to F is given, therefore the ratio of D to F is given.

There-

Therefore, if two, or any number of magnitudes, have a given ratio to one another, and if these have a given ratio to a like number of other magnitudes, each to each, all these other magnitudes have a given ratio to one another; which was to be demonstrated.

PROPOSITION X.

If a magnitude be greater than in a given ratio to another magnitude, by a given magnitude, the sum of the two will be greater than in a given ratio to the same magnitude, by a given magnitude. And if the two, taken together, be greater than in a given proportion to the same magnitude, by a given magnitude; then will the remainder be either greater than in a given ratio to the same magnitude, by a given magnitude; or the remainder, together with the consequent, to which that same magnitude has a given proportion, is a given magnitude.

Let AB be greater than in a given ratio to BC, by a given magnitude, which let be AD.

I say that AC, the sum of the two, will be greater than in a given ratio to BC, by a given magnitude. For since AB is greater than in a given ratio to BC, by the given magnitude AD, take away AD, then will the ratio of DB

to BC be given^p, consequently^q the ratio of DC to BC is given; but AD is given in magnitude,

^p Def. 11. of this.

^q Dat. 6.

nitude, therefore AC is greater than in a given ratio to BC, by the given magnitude AD^r. ^r Def. 11. of this.

Again, let the whole AC be greater than in a given ratio to BC, by the given magnitude AD. I say the remainder

AB will either be greater $\begin{array}{ccccccc} A & & D & & B & & C \\ & \text{---} & | & \text{---} & | & \text{---} & \end{array}$ than in a given ratio to

BC, by a given magnitude; or AB, together with the consequent, to which BC has a given ratio, is a given magnitude. For the given magnitude AD must either be smaller than AB, or greater than AB. And first let it be smaller than AB; take it away, then will the ratio of

DC to CB be given^f, and consequently ^t the ^f Def. 11. of this.

ratio of DC to DB is given; therefore ^v the ^t Dat. 5.

ratio of DB to BC is given; wherefore AB is ^v Dat. 8.

greater than in a given ratio to BC, by the given magnitude AD^u. But if the given magnitude ^u Def. 11. of this.

AD be greater than AB, take away AD; then is the ratio $\begin{array}{ccccccc} A & & B & & D & & C \\ & \text{---} & | & \text{---} & | & \text{---} & \end{array}$ of DC to BC given^w, and by

inversion the ratio of BC to CD is given; where- ^w Def. 11. of this.

fore ^x the ratio of BC to BD is given, that is the ^x Dat. 5.

remainder AB, together with the consequent BD, to which BC has a given proportion, is a given magnitude.

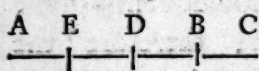
Therefore, if a magnitude be greater than in a given ratio to another magnitude, by a given magnitude, &c. which was to be demonstrated.

PROPOSITION XI.

If a magnitude be greater than in a given ratio to another magnitude, by a given magnitude; the same magnitude will be greater than in a given ratio to the sum of the two, by a given magnitude; and if that magnitude be greater than in a given ratio to the sum of the two by a given magnitude; it will also be greater than in a given ratio to the remainder by a given magnitude.

Let AB be greater than in a given ratio to BC, by a given magnitude as AD: I say the

same magnitude AB will be greater than in a given ratio to the whole AC, by a given



magnitude; because AB is greater than in a given ratio to BC, by the given magnitude AD; take away AD, then is the ratio of DB

^y Def. 11. to BC given^y, and by inversion, the ratio of
of this. BC to BD is given, consequently ^z the ratio of
^z Dat. 6. DC to DB is given; and as DC is to DB, so
let AD be to DE; but AD is given, wherefore

^a Dat. 2. ^a DE is given, and consequently AE ^b is given.

^b Dat. 4. And because DC is to DB, as AD is to DE,

^c Prop. 12. therefore ^c as DC is to DB, so is AC to EB,

Euc. lib. 5. wherefore the ratio of AC to EB is given; and

by inversion, the ratio of EB to AC is given;

but AE is a given magnitude, therefore the

magnitude AB is greater than in a given ratio

^d Def. 11. to the whole AC, by a given magnitude^d.
of this.

Again,

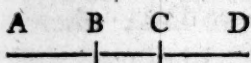
Again, let AB be greater than in a given ratio to AC, by the given magnitude AE. I say the same magnitude AB, is greater than in a given ratio to the remainder BC, by a given magnitude. For because AB is greater than in a given ratio to AC, by the given magnitude AE; take away the given magnitude AE, then is the ratio of EB to AC given^e, * Def. 11. and consequently the ratio of AC to EB is given. of this. As AC is to EB, so let AD be to DE; wherefore the ratio of AD to DE is given, and ^f therefore the ratio of AD to AE is given; and ^f Dat. 5. by inversion the ratio of AE to AD is given; but AE is given; therefore ^g AD is given, ^g Dat. 2. and consequently ED ^h is given. Again, because AC is to EB, as AD is to DE, therefore ⁱ as AC is to EB, so is the remainder DC, to ⁱ Prop. 19. the remainder DB, consequently the ratio of ^{Euc. lib. 5.} DC to DB is given; wherefore ^k the ratio of DC ^k Dat. 5. to BC is given, and hence the ratio of BD to BC is given^l, but AD is given; wherefore AB ^l Dat. 8. is greater than in a given ratio to the remainder BC by the given magnitude AD.

Therefore, if a magnitude be greater than in a given ratio to another magnitude, by a given magnitude, &c. which was to be demonstrated.

PROPOSITION XII.

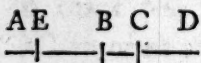
If there be three such magnitudes, and if the sum of the first and second be given, and the sum of the second and third be also given; then will the first be either equal to the third, or one of them will exceed the other by a given magnitude.

Let AB, BC, and CD, be three magnitudes, and let AC, the sum of the first and second, be given, and BD, the sum of the second and third be given. I say the first AB is



either equal to the third CD, or one of them exceeds the other by a given magnitude; for AC must be either equal to BD, or not; first let AC be equal to BD, take away BC from both, and there will remain AB equal to CD.

* Dat. 4.



Again let AC be not equal to BD; let AC be the greatest, cut a part of it as EC, equal to BD; then because BD is given, therefore EC, which is equal to it, is also given; but AC is given, wherefore CE is given^m, and consequently AC exceeds BD, by the given magnitude AE.

Therefore, if there be three magnitudes, &c. which was to be demonstrated.

PROPOSITION XIII.

If there be three magnitudes, and if the first has a given ratio to the second, but the second is more than in a given proportion to the third, by a given magnitude; then will the first be greater than in a given ratio to the third, by a given magnitude.

Let AB, CD, and E, be three magnitudes, and let AB have a given ratio to CD; but CD is greater than in a given ratio to E, by the given magnitude CF. I say the first AB is also greater than in a given ratio to the third E, by a given magnitude. For since the ratio of AB to CD is given, the ratio of CD to AB is given, and CF



is given, let it be as CD is to AB, so is CF to AG; then is the ratio of CF to AG given; but CF is given, therefore ⁿ AG is given, ^a Dat. 2. Again, because CD is greater than in a given proportion to E by CF, take away CF, and the ratio of FD to E is given^o. And because CD [•] Def. 11. is to AB, as CF is to AG, therefore FD will ^{of this.} be to GB, as CD is to AB^p; but the ratio of ^p Prop. 19. CD to AB is given, therefore the ratio of FD ^{Euc. lib. 5.} to GB is given, and consequently the ratio of GB to FD is given; but the ratio of FD to E is given, wherefore ^q the ratio of GB to E is ^a Dat. 8. given; but AG is a given magnitude, where-

C

fore

fore the first AB is greater than in a given ratio to the third E, by a given magnitude^r.
^r Def. 11. of this.

Therefore, if there be three magnitudes, and if the first has a given ratio to the second, but the second is more than in a given ratio to the third, by a given magnitude; then will the first be greater than in a given ratio to the third, by a given magnitude; which was to be demonstrated.

PROPOSITION XIV.

If any two magnitudes have a given ratio to one another, and if given magnitudes be added to each of them; the whole will either have a given proportion to the whole, or one of them will be more than in a given ratio to the other, by a given magnitude.

Let AB and CD be two magnitudes, which have a given ratio to one another, and let the
 $\begin{array}{ccccccc} \text{A} & \text{B} & & \text{G} & \text{E} & & \\ \hline & | & \text{---} & | & & & \\ \text{C} & & \text{D} & & \text{F} & & \\ \hline & & | & & & & \end{array}$ given magnitudes BE, DF,
 be added to them, each to each. I say the whole AE
 will either have a given ratio

to the whole CF, or one of them will be greater than in a given ratio to the other, by a given magnitude. For, since the magnitudes BE and DF are each given, the ratio of BE to DF is given^r, and the ratio of AB to CD is given. Now the ratio of AB to CD must either be equal to the ratio of BE to DF, or not. First, let them be equal; then, because AB is to CD, as
 BE

BE is to DF, it will be^t, as AB is to CD, so ^{Prop. 12.}
 is AE to BF; but the ratio of AB to CD is ^{Euc. lib. 5.}
 given, therefore the ratio of the whole AE to
 the whole CF is given. But if BE be not to
 DF as AB is to CD, let AB be to CD as BG is
 to DF; but the ratio of AB to CD is given,
 therefore the ratio of BG to FD is given; but
 DF is given, wherefore BG is given^v; but BE ^{Dat. 2.}
 is given, therefore GE is given^w. Again, be- ^{Dat. 4.}
 cause AB is to CD, as BG is to DF, therefore,
^x as AB is to CD, so is AG to CF; but the ratio ^{Prop. 12.}
 of AB to CD is given, therefore the ratio of ^{Euc. lib. 5.}
 AG to CF is given; but GE is given, there-
 fore ^y the whole AE is greater than in a given ^{Def. 11.}
 ratio to CF, by the given magnitude GE. ^{of this.}

Therefore, if any two magnitudes have a given ratio to one another, and if given magnitudes be added to each of them; the whole will either have a given proportion to the whole, or one of them will be more than in a given ratio to the other, by a given magnitude; which was to be demonstrated.

PROPOSITION XV.

If any two magnitudes have a given ratio to one another, and from them given magnitudes be taken away, the remainders will either have a given ratio to one another, or one of them will be greater than in a given proportion to the other, by a given magnitude.

EUCLID'S DATA.

Let AB and CD be two magnitudes, which have a given ratio to one another, and let the



two given magnitudes AE and CF be taken away; I say the remainder EB has either a given ratio to the remainder FD, or the one is

greater than in a given ratio to the other, by a given magnitude. For AE must either have the same ratio to CF that AB has to CD, or not. And first let it be the same. Then because AB is to CD, as AE is to CF, the residue EB will be to the residue FD, as the

^a Prop. 19.
Euc. lib. 5.

whole AB is to the whole CD; ^a but the ratio of AB to CD is given, therefore the ratio of the remainder EB to the remainder FD is given.

Again, let AE have not the same ratio to CF, that AB has to CD. Then let AB be to CD, as AG is to CF, wherefore the ratio of AG to CF is given; but CF is given, therefore AG is

^a Dat. 2.

given^a; but AE is given, wherefore EG is

^b Dat. 4.

given^b. Again, because AB is to CD, as AG is to FC, therefore the remainder GB is to the remainder FD, as the whole AB is to the whole

^c Prop. 19.
Euc. lib. 5.

CD^c; but the ratio of AB to CD is given, wherefore the ratio of GB to FD is given; but EG is given, wherefore EB is greater than in a given ratio to FD, by a given magnitude.

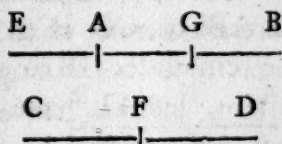
Therefore, if two magnitudes have a given ratio to one another, and from them given magnitudes be taken away, the remainders will either have

have a given ratio to one another, or one of them will be greater than in a given proportion to the other, by a given magnitude; which was to be demonstrated.

PROPOSITION XVI.

If any magnitude has a given proportion to another, and if to the one a given magnitude be added, and a given magnitude subtracted from the other, that sum will be more than in a given ratio to the remainder, by a given magnitude.

Let AB and CD be two magnitudes, which have a given ratio to one another, and to AB let the given magnitude AE be added, and from CD the given magnitude CF subtracted; I say that EB will be more than in a



given ratio to FD, by a given magnitude. For as AB is to CD, so let AG be to CF. Then because the ratio of AB to CD is given, therefore the ratio of AG to CF is given; but CF is given, therefore ^d AG is given; but EA is ^a Dat. 2. given, therefore EG ^e is given. Again, because ^e Dat. 3. AB is to CD, as AG is to CF, wherefore ^f Prop. 19. GB is to FD, as AB to CD; but the ratio of ^f Euc. lib. 5. AB to CD is given, wherefore the ratio of GB to FD is given; but EG is given, consequently ^g EB is more than in a given ratio to FD, by ^g Def. 11. the given magnitude EG. of this.

C 3

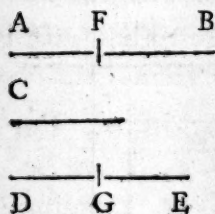
Therefore,

Therefore, if any magnitude has a given proportion to another, and if to the one a given magnitude be added, and a given magnitude subtracted from the other, that sum will be more than in a given ratio to the remainder, by a given magnitude; which was to be demonstrated.

PROPOSITION XVII.

If two magnitudes be each more than in a given ratio to a third, by a given magnitude, these magnitudes will either have a given ratio to one another, or the one will be more than in a given ratio to the other, by a given magnitude.

Let AB and DE be two magnitudes, which are each more than in a given ratio to the third magnitude C, by the given magnitudes AF and DG; I say AB has either a given ratio to DE, or one of them is more than in a given ratio to the other, by a given magnitude. Because AB



C, by AF, take away AF, then is the ratio of

^b Def. 11, FB to C given^h, for the same reason the ratio of GE to C is given. Then because the ratios of FB to C, and GE to C, are each given,

ⁱ Dat. 8, whereforeⁱ the ratio of FB to GE is given.

Since the ratio of FB to GE is given, and to them are added the given magnitudes AF and

^k Dat. 14, DG, wherefore^k the whole AB has either a given

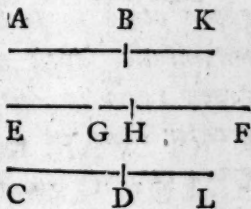
given ratio to the whole DE, or the one is more than in a given ratio to the other, by a given magnitude.

Therefore, if two magnitudes be each more than in a given ratio to a third, by a given magnitude, these magnitudes will either have a given ratio to one another, or the one will be more than in a given ratio to the other, by a given magnitude; which was to be demonstrated.

PROPOSITION XVIII.

If one magnitude be more than in a given ratio to two other magnitudes, by a given magnitude, these magnitudes will either have a given ratio to one another, or the one will be greater than in a given ratio to the other, by a given magnitude.

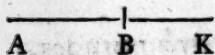
Let AB, CD, be two magnitudes, and EF a third, and let EF be more than in a given ratio to AB and CD, by the given magnitudes EG, EH; I say that AB, CD, will either have a given ratio to one another, or the one will be more than in a given ratio to the other, by a given magnitude.



For since EF is greater than in a given ratio to AB, by EG, take away EG, then will the ratio of GF to AB be given^k. For the same^k Def. 11. reason the ratio of HF to CD is given. Let^{of this.} GF be to AB, as EG is to BK; and as HF

is to CD, so let EH be to DL. Then because the ratio of GF to AB is given, therefore the

¹ Dat. 2.

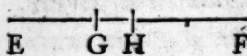


ratio of EG to BK is given;

but EG is given, therefore ¹ BK is given. And since GF

^m Prop. 12.

Euc. lib. 5.



is to AB, as EG is to BK, wherefore ^m as GF is to AB,

so is EF to AK; but the ratio of GF to AB is given, where-

fore the ratio of EF to AK is given. For the

same reason the ratio of EF to CL is given,

^a Dat. 8.

wherefore ⁿ the ratio of AK to CL is given;

but KB and DL are each given magnitudes;

hence from AK, CL, which have a given ratio

to one another, given magnitudes BK, DL, are

^o Dat. 15.

taken away, wherefore ^o the remainders AB,

CD, will either have a given ratio to one another,

or the one is more than in a given ratio to

the other, by a given magnitude.

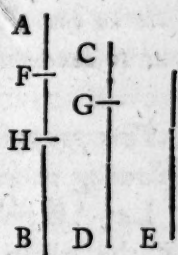
Therefore, if one magnitude be more than in a given ratio to two other magnitudes, by a given magnitude, these magnitudes will either have a given ratio to one another, or the one will be greater than in a given ratio to the other, by a given magnitude; which was to be demonstrated.

PROPOSITION XIX.

If there be three magnitudes, and if the first be greater than in a given ratio to the second, by a given magnitude, and the second is greater than in

a given ratio to the third, by a given magnitude, the first will also be greater than in a given ratio to the third, by a given magnitude.

Let AB, CD, and E, be three magnitudes, and let the first AB be greater than in a given ratio to the second CD, by the given magnitude AF, and let the second CD be greater than in a given ratio to the third E, by the given magnitude CG; I say the first magnitude AB is more than in a given ratio to the third E, by a given magnitude. For since AB is more than in a given ratio to CD, by the magnitude AF, take away AF, and the ratio of FB to CD is given^p. Also, because CD is more than in a given ratio to E, by the given magnitude CG, therefore the ratio of GD to E is given. Let FB be to CD as FH is to CG, then is the ratio of FH to CG given; but CG is given, therefore FH is given^q; but AF is given, ^q Dat. 2. wherefore^r AH is given. Again, because FB ^r Dat. 3. is to CD, as FH is to CG, therefore^s HB is ^s Prop. 19. to GD, as FB is to CD; but the ratio of FB ^t Euc. lib. 5. to CD is given, wherefore the ratio of HB to GD is given; but the ratio of GD to E is given, wherefore^t the ratio of HB to E is given; but ^t Dat. 8. AH is a given magnitude, consequently the first AB is more than in a given ratio to the third E, by the given magnitude AH.



Therefore,

Therefore, if there be three magnitudes, and if the first be more than in a given ratio to the second, by a given magnitude, and the second is greater than in a given ratio to the third, by a given magnitude, the first will also be greater than in a given ratio to the third, by a given magnitude; which was to be demonstrated.

This proposition is demonstrated otherwise in the following manner.

Let AB be more than in a given ratio to CD, by the given magnitude AF, take away AF, and the ratio of FB to CD is given^u. Then because FB has a given ratio to CD, and CD is more than in a given ratio to E, by a given magnitude; therefore, the first FB to the third E, will be more than in a given ratio, by a given magnitude, which let be

^u Def. 11.
of this.

^v Dat. 13.

^w Def. 11.
of this.

^x Dat. 3.

^y Def. 11.
of this.

FG, take away FG, and ^w the ratio of GB to E is given; but because AF is given, and ^x FG given, therefore the whole AG ^x is given, wherefore the first AB is more than in a given ratio to the third E, by a given magnitude.^y

Therefore, if there be three magnitudes, and if the first be more than in a given ratio to the second, by a given magnitude, and the second is greater than in a given ratio to the third, by a given magnitude, the first will also be greater than in a given ratio to the third, by a given magnitude; which was to be demonstrated.

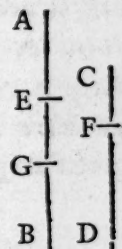
P R O.

PROPOSITION XX.

If from two given magnitudes, two magnitudes, which have a given ratio to one another, be taken away, the remainders will either have a given ratio to one another, or the one will be more than in a given ratio to the other, by a given magnitude.

Let AB and CD be two given magnitudes, and let AE and CF, which have a given ratio

to one another, be taken from them; I say the remainder EB, will either have a given ratio to the remainder FD, or the one will be more than in a given ratio to the other, by a given magnitude. For since AB and CD are given, the ratio of AB to CD is



given^y, wherefore the ratio of AB to CD will ⁷ Dat. 1. either be equal to the ratio of AE to CF, or not. And first let them be equal: then because AB is to CD, as the part AE to the part CF, the remainder EB is to the remainder FD, as AB is to CD^z; but the ratio of AB to CD is ^z Prop. 19. given, consequently the ratio of EB to FD is ^{Euc. lib. 5.} given.

But if AB be not to CD, as AE is to CF, let AG be to CD, as AE is to CF; but the ratio of AE to CF is given^a, wherefore the ratio ^a Hyp. of AG to CD is given; but CD is a given magnitude, therefore AG^b is given; but the whole ^b Dat. 2. AB is given, consequently GB^c is given. A- ^c Dat. 4. gain,

^d Prop. 19.
 Enc. lib. 5.

^e Def. 11.
 of this.

gain, because AG is to CD, as AE is to CF, wherefore^d EG is to FD, as AG is to CD; but the ratio of AG to CD is given, wherefore the ratio of EG to FD is given; but GB is a given magnitude, therefore^e the remainder EB is more than in a given ratio to the remainder FD, by the given magnitude GB.

Therefore, if from two given magnitudes, two magnitudes, which have a given ratio to one another, be taken away, the remainders will either have a given ratio to one another, or the one will be more than in a given ratio to the other, by a given magnitude; which was to be demonstrated.

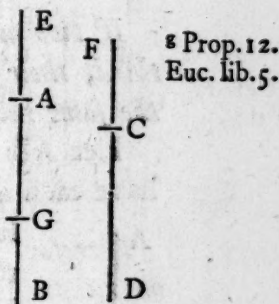
PROPOSITION XXI.

If to two given magnitudes, magnitudes be added, having a given ratio to one another, the sums will either have a given ratio to one another, or the one will be more than in a given ratio to the other, by a given magnitude.

Let AB and CD be two given magnitudes, and to them let AE, CF, having a given ratio to one another, be added; I say the whole EB will either have a given ratio to the whole FD, or the one will be more than in a given ratio to the other, by a given magnitude. For since AB and CD are given, the ratio of AB to CD is given^f. Then will the ratio of AB to CD be

^f Dat. 1.

be either equal to the ratio of AE to CF, or not. First let them be equal: then because AB is to CD, as AE is to CF, wherefore^s AB is to CD, as EB is to FD; but the ratio of AB to CD is given, wherefore the ratio of EB to FD is given.



But if AB be not to CD, as AE to FC, let AG be to CD, as AE to FC; but the ratio of EA to FC is given^h, wherefore the ratio of AG to CD is given; but CD is a given magnitude, thereforeⁱ AG is given; but AB is given, therefore the remainder GB is given^k. Again, because EA is to FC, as AG is to CD, wherefore^l as EA is to FC, so is EG to FD; but the ratio of EA to FC is given^m, therefore the ratio of EG to FD is given; but GB is a given magnitude, whereforeⁿ the whole EB is more than in a given ratio to the whole FD, by the given magnitude GB.

Therefore, if to two given magnitudes, magnitudes be added, having a given ratio to one another, the sums will either have a given ratio to one another, or the one will be more than in a given ratio to the other, by a given magnitude; which was to be demonstrated.

PROPOSITION XXII.

If two magnitudes have each a given ratio to a third, their sum will also have a given ratio to the same third.

Let AB and BC be two magnitudes, which have each a given ratio to a third D; I say their

A

B

C

D

• Hyp.
• Dat. 8.

• Dat. 6.

• Dat. 8.

sum AC will also have a given ratio to D. For since the ratio of AB to D is given, and the ratio of BC to D is likewise given^o, therefore^p the ratio of AB to BC is given, and consequently^q the ratio of AC to BC is given; but the ratio of BC to D is given, wherefore^r of AC to D is given.

Therefore, if two magnitudes have each a given ratio to a third, their sum will also have a given ratio to the same third; which was to be demonstrated.

PROPOSITION XXIII.

If a whole magnitude has a given ratio to another whole, and the parts of the one have also given ratios to the parts of the other, each to each, though they are not the same ratios with the former, then will the parts have each given ratios to their respective wholes, and to one another, and all to all.

Let the whole AB have a given ratio to the whole CD, and let the part AE have a given ratio
to

to the part CF, and the part EB to the part FD, though not the same ratio with that of AE to CF; I say the parts AE, EB, will each have a given ratio to the whole AB, and to each other, and all will have a given ratio to all. For let AE be to CF, as AG is to CD; but the ratio of AE to CF is given^f, wherefore the ratio of AG to CD is given; but the ratio of AB to CD is given,



therefore^t the ratio of AG to AB is given, and^t Dat. 8. consequently^v the ratio of AB to GB is given.^v Dat. 5. Again, because AE is to CF, as AG is to CD, therefore^w AE is to CF, as EG is to FD; but^w Prop. 19. Euc. lib. 5. the ratio of AE to CF is given, wherefore the ratio of EG to FD is given; but the ratio of EB to FD is given, therefore^x the ratio of EG^x Dat. 8. to EB is given, and consequently^y the ratio of^y Dat. 5. EB to BG is given; but the ratio of BG to AB is given, therefore^z the ratio of EB to AB is given,^z Dat. 8. and consequently^a the ratio of AE to EB is^a Cor. Dat. given. Again, since the ratio of AE to AB⁵ is given, and the ratio of AB to CD is given, therefore the ratio of AE to CD is given; but the ratio of AE to CF is given, therefore^b the^b Dat. 8. ratio of CD to CF is given, &c.

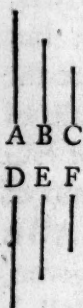
Therefore, if a whole magnitude has a given proportion to another whole magnitude, and the parts of the one have also given ratios to the parts of the other, each to each, though they are not the same ratios with the former, then will each of the

parts have given ratios to their respective wholes, and to one another, and all to all, &c. which was to be demonstrated.

PROPOSITION XXIV.

If there be three right lines in continued proportion, and if the ratio of the first to the third be given, the ratio of the first to the second is also given.

Let A, B, and C, be three right lines in continued proportion, and let the ratio of the first A to the third C be given; I say the ratio of the first A to the second B is also given. For let D be a line of a given magnitude; find a fourth proportional to A; C, and D, which let be F^c. Then because the ratio of A to C is given^d, therefore the ratio of D to F is given; but D is given in magnitude, wherefore F is given in magnitude^e. Then



^c Prop. 12.
Euc. lib. 6.

^d Hyp.

^e Dat. 2.

^f Prop. 13.
Euc. lib. 6.

^g Prop. 17.
Euc. lib. 6.

^h Def. 1.
of this.

ⁱ Dat. 1.
^k Prop. 1.
Euc. lib. 6.

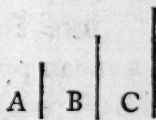
find a mean proportional to the two given lines D and F, which let be E^f. Then because D is to E, as E is to F, wherefore the rectangle contained between D and F is equal to the square of E^g; but the rectangle contained between D and F is given because the two lines D and F are each given in magnitude, therefore^h the square of E is given, and consequently the right line E is given in magnitude; but D is given, therefore the ratio of D to E is givenⁱ. Again^k,

as

as D is to F, so is the square of D to the rectangle contained between D and F; but the rectangle contained between D and F is equal to the square of E, therefore as D is to F, so is the square of D to the square of E; but as D is to F, so is A to C, therefore as A is to C, so is the square of D to the square of E. For the same reason as before, it will be as A is to C, so is the square of A to the square of B; wherefore^l as the square of A is to the square of B, ^lProp. 11. Euc. lib. 5. so is the square of D to the square of E, and consequently^m as A is to B, so is D to E; but ^mProp. 22. Euc. lib. 6. the ratio of D to E is given, wherefore the ratio of the first A to the second B is given.

Therefore, if there be three right lines in continued proportion, and if the ratio of the first to the third be given, the ratio of the first to the second is also given; which was to be demonstrated.

This proposition is otherwise demonstrated in the following Manner: Let A, B, and C, be three lines in continued proportion, and let the ratio of A to C be given; I say the ratio of A to B is also given, because A is to C as



the square of A is to the rectangle contained between A and Cⁿ; but the rectangle contained between A and C is equal to the square of B^o, ⁿProp. 1. Euc. lib. 6. therefore the square of A is to the square of B, ^oProp. 17. Euc. lib. 6. as A is to C; but the ratio of A to C is given, therefore the ratio of the square of A to the square of B is given; wherefore the ratio of the

D

line

line A to the line B is given, because the ratio of their squares are given.

Note, This last demonstration is neither elegant nor just; for the concluding step depends upon that principle, viz. if the ratio of the squares of any two lines be given, the ratio of the lines themselves is also given, which is not proved in any of the antecedent data; and therefore this demonstration (though used by Gregory in his Edit. of Euclid's works in Greek and Latin, and Claudius Hardy in his Greek and Latin Edition of the Data) is not conclusive.

PROPOSITION XXV.

If any two lines given in position, mutually cut one another, the point of intersection is given.



Let the two lines AB, CD, given in position, mutually cut one another at E; I say the point of intersection, E, is given.

For if the point E is not given, then one of the lines, AB or CD, must change its place; but by Hyp. they are both given in position, and cannot change their situation, therefore the point of intersection, E, cannot change its position, consequently ^p the point E is given.

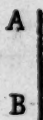
Def. 4.
of this.

Therefore, if any two lines given in position, mutually cut one another, the point of intersection is given; which was to be demonstrated.

PROPOSITION XXVI.

A right line, joining two given points, is given both in position and magnitude.

Let A and B be two given points; I say the right line AB, which joins them, is given both in magnitude and in position. For if, while the point A retains its place, the line AB should change either its position or magnitude, then the point B will also change its position, contrary to the hypothesis, which is absurd; therefore the right line AB is given both in position and magnitude.

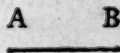


Therefore, if two points be given, a right line joining them is given both in magnitude and position; which was to be demonstrated.

PROPOSITION XXVII.

If a right line be given in position and magnitude, and one of its extremities be given, the other extremity is also given.

Let AB be a right line, given in position and magnitude, one of whose extremities, as A, is given; I say the other extremity B is also given. For if while the point A retains its situation, the point B should change its place; then the line AB will change either its position or magnitude, or both, which is contrary to the hypothesis, and absurd; wherefore the point B is given.



This proposition is more elegantly demonstrated in the following manner.

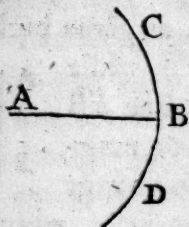
About the center A, with the distance of the given line AB, describe the circle CBD; then because the center of the circle CBD is given, and its radius AB is given in magnitude^a, wherefore the circumference CBD is given in position^r; but the line AB is given in position^t, wherefore^t the point of intersection B is given.

^a Hypoth.

^r Def. 6.
of this.

^t Hypoth.

^t Dat. 25.



Therefore, if a right line be given in position and magnitude, and if one of its extremities be given, the other extremity is also given; which was to be demonstrated.

PROPOSITION XXVIII.

If through a given point a right line be drawn, in a contrary position, or parallel to a right line given in position, that right line is also given in position.

Let A be a given point, and BC a right line given in position, and through the point A let the right line DE be drawn parallel to BC; I say the right line DE is given in position. For if DE is not given in position, while the point A retains its place, let DE change its position to



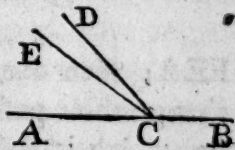
to FG: then because BC is parallel to FG^v, ^v Hypoth. and BC is also parallel to ED, wherefore DE is parallel to FG^w, and at the same time cut^w ^{Prop. 30. Euc. lib. 1.} each other, which is absurd; wherefore DE cannot change its place, and consequently is^x given in position. ^x Def. 4. of this.

Therefore, if through a given point a right line be drawn, parallel to a right line given in position, that right line is also given in position; which was to be demonstrated.

PROPOSITION XXIX.

If from a given point in a right line given in position a right line be drawn, making a given angle with it, that right line is also given in position.

Let AB be a right line given in position, and C a given point in it, and from the point C let CD be drawn, making a given angle with the line AB; I say CD is given in position. For if CD is not given in position, while the point C retains its place, let the line CD change its position to EC; but the magnitude of the angle DCA is given^y, and therefore will always be^y ^{Hypoth.} the same, consequently the angle DCA is equal to the angle ECA, a part to the whole, which



* Def. 4.
of this.

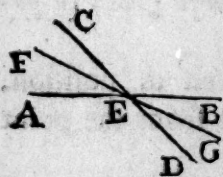
is absurd; wherefore the right line DC cannot change its place, and therefore * it is given in position.

Therefore, if from a given point in a right line given in position, a right line be drawn, making a given angle with it, that right line is also given in position; which was to be demonstrated.

L E M M A.

If two lines, given in position, mutually cut one another, the angles formed by them are given in magnitude.

Let the two lines AB CD be given in position, and mutually cutting one another at the point



E; I say the angles CEA CEB are of a given magnitude. For if the angle CEA is not of a given magnitude, CEA may alter its magnitude, which let be to that of

FEA; then one of the lines CD or AB will change their position, contrary to the hypothesis, which is absurd; therefore the angle CEA cannot change its magnitude, and consequently is of a given magnitude.

P R O.

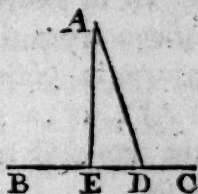
PROPOSITION XXX.

If from a given point, out of a right line given in position, a right line be drawn, making a given angle with it, that right line is also given in position.

This proposition is demonstrated four different ways.

FIRST.

Let A be the given point, BC the right line given in position, and from the point A let AD be drawn, making a given angle with the line BC; I say the line AD is given in position. For if AD is not given in position, while the point A retains its place, let AD change



its position to AE; but it will still make the same angle with the line BC, because that angle is given in magnitude^a; wherefore the angle AEB^a Hypoth. is equal to the angle ADE; hence the triangle ADE has its outward angle AEB equal to its inward opposite angle ADE, which is absurd^b; ^b Prop. 16. Euc. lib. 1. wherefore AD cannot change its place, and consequently^c AD is given in position. ^c Def. 4. of this.

Therefore, if from a given point, out of a right line given in position, a right line be drawn, making a given angle with it, that right line is

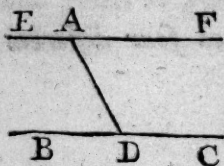
EUCLID'S DATA.

also given in position; which was to be demonstrated.

SECOND.

Through the given point A draw the right line EF, parallel to BC^d; then because the line EF

^d Prop. 31.
Euc. lib. 1.



^e Prop. 29.
Euc. lib. 1.

is parallel to BC, and cut by the line AD, the angle FAD is equal to ADB^e; but ADB is given, therefore the angle FAD is given. And because the line EF is drawn through

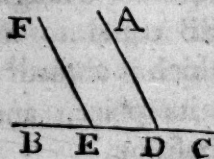
the given point A, parallel to the line BC, given in position, wherefore^f the line EF is given in position, and hence the line AD is drawn from the given point A, in the line EF given in position, making with it the angle

^g Dat. 29. FAD, of a given magnitude, consequently^g the line AD is given in position.

THIRD.

Take any point in the line BC, as E, and through the point E draw the line EF, parallel

^h Prop. 31.
Euc. lib. 1.



ⁱ Prop. 29.
Euc. lib. 1.

to AD^h. Then because AD is parallel to FE, and cut with the line BC, the angle FEB is equal to the angle ADBⁱ; but ADB is given in magnitude^k, wherefore the angle

^k Hypoth.

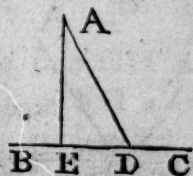
^l Dat. 29. FED is given in magnitude, consequently^l the line FE is given in position; wherefore the line

line AD is drawn through the given point D, parallel to FE, given in position, therefore^m theⁿ Dat. 28. line AD is given in position; *which was to be demonstrated.*

FOURTH.

Take any point in the line BC, as E, join the line EA; then because the two points A and E are both given, where-

foreⁿ the right line AE is given both in position and magnitude; but BC is given in position^o, wherefore the angle AED is given in mag-



ⁿ Dat. 26.

^o Hypoth.

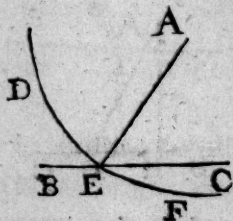
nitude^p; but the angle ADE is given in mag-^p Lemma preceding.
nitude, therefore^q the remaining angle EAD is^q Prop. 32.
given in magnitude, wherefore the line AD is^q Euc. lib. 1.
drawn from the given point A in the line AE, & Dat. 4.
given in position, making with it an angle, as
EAD, of a given magnitude; therefore^r AD^r Dat. 29.
is given in position.

Therefore, if from a given point, out of a right line given in position, a right line be drawn, making a given angle with it, that right line is also given in position; which was to be demonstrated.

PROPOSITION XXXI.

If a right line, given in magnitude, be drawn from a given point, cutting a right line given in position, that right line is given in position.

Let A be a given point, and BC a right line given in position, and from the point A let the line AE, given in magnitude, be drawn, cutting the line BC at E; I say the line AE is given in position. For about the center A, with the distance of the given line AE, describe the circle DEF: then because A,



the center of the circle DEF, is given, and its radius is given in magnitude, therefore ^t the circumference DEF is given in position; but ^t Def. 6. of this. ^t Hypoth. the line BC ^t is given in position, wherefore ^v ^v Dat 25. the point of intersection, E, is given; but the ^v ^v Dat. 26. point A is given, wherefore ^w the line AE is given in position.

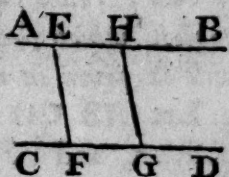
Therefore, if from a given point a right line, given in magnitude, be drawn, cutting a right line given in position, that right line is given in position; which was to be demonstrated.

PROPOSITION XXXII.

If a right line be drawn, cutting and bounded by two parallel right lines which are given in position,

position, and making given angles with them, that right line is given in magnitude.

Let AB CD be two parallel right lines given in position, and let the right line EF be drawn cutting them, so that the angles BEF EFD are given in magnitude; I say the right line EF is given in magnitude. For take any point,



as G , in the line CD , and through the point G draw the line GH parallel to EF ^a, then is the Figure EG a parallelogram; and because HG is parallel to EF , and cut by the line CD , the angle HGD is equal to the angle EFG ^b; but the angle EFG is given, wherefore the angle HGD is given, and therefore^c the line GH is given in position; but AB is given in position, therefore the point H ^a is given, consequently HG ^a is given in magnitude; but HG is equal to EF ^b, wherefore the line EF is given in magnitude.

^a Prop. 31.
Euc. lib. 1.

^b Prop. 29.
Euc. lib. 1.

^c Dat. 29.

^a Dat. 25.

^b Dat. 26.

^c Prop. 34.
Euc. lib. 1.

Therefore, if a right line be drawn, cutting and bounded by two parallel right lines which are given in position, and making given angles with them, that right line is given in magnitude; which was to be demonstrated.

PROPOSITION XXXIII.

If a right line, given in magnitude, be bounded by two parallel right lines given in position, the angles, which that line makes with the parallels, will be given in magnitude.

Let AB CD be two parallel lines given in position, and EF a right line given in magnitude, and bounded by the parallels; I say the line EF cuts the two parallel lines AB CD in given angles; that is, the angles BEF EFD are each given in magnitude. For take any point in the line AB , as G , and from G draw GH parallel



to EF^d , then is the figure EH a parallelogram, and $^e GH$ is equal to EF ; but EF is given in magnitude^f, therefore GH is given in magnitude. About the center G , with the distance GH , describe the circle KHL ; then because the center G is given, and the radius GH given in magnitude, the circumference KHL is given in position^g; but CD is given in position, therefore^h the point H is given; but G is given, therefore the line GH is given in positionⁱ, wherefore^k the angle GHD is given in magnitude; but the angle GHD is equal to the angle EFD^l , therefore the angle EFD is given in magnitude, and consequently^m the angle BEF is given in magnitude.

Therefore,

^d Prop. 31.

Euc. lib. 1.

^e Prop. 34.

Euc. lib. 1.

^f Hypoth.

^g Def. 6.

of this.

^h Dat. 25.

ⁱ Dat. 26.

Lemma to

Dat. 30.

^k Prop. 29.

Euc. lib. 1.

^l Prop. 29.

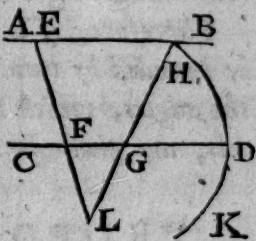
Euc. lib. 1.

& Dat. 4.

Therefore, if a right line, given in magnitude, be bounded by two parallel right lines given in position, the angles, which that line makes with the parallels, will be given in magnitude; which was to be demonstrated.

This proposition is also demonstrated in the following manner.

Take any point in the line CD, as G, and cut a Part of it, as GD, equal to the line EF, which is given in magnitude; about the center G, with the distance GD, describe the circle HDK; which cuts the line AB in H, and join HG; then because EF is given in magnitude, and GD is equal to it, therefore GD is given in magnitude; and the point G is given, therefore ^m the circumference ⁿ HDK is given in position; but AB is given in positionⁿ, therefore the point H is given^o; but G is given, wherefore ^p HG is given in position, and consequently the angle HGD is given in magnitude^q. If the line HG is parallel to EF, the angle HGD is equal to EFG^r, therefore the remaining angle FEB^r is given. But if the line HG is not parallel to EF, they will, when produced, cut one another, which let be in the point L; then because FG is parallel to EH^t, as EF is to FL, so is HG to GL; but ^t Prop. 2. ^{FE} Euc. lib. 6.



^m Def. 6.
ⁿ of this.

^o Hypoth.
^p Dat. 25.
^q Dat. 26.

^r Lemma,
^s Dat. 30.
^t Prop. 29.
Euc. lib. 1.
^u Prop. 29.
Euc. lib. 1.
& Dat. 4.

Prop. 14.
 Euc. lib. 5.

Prop. 5.
 Euc. lib. 1.

Prop. 15.
 Euc. lib. 1.

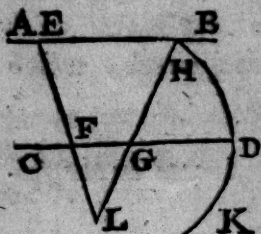
Prop. 13.
 Euc. lib. 1.

& Dat. 4.

Prop. 29.

Euc. lib. 1.

& Dat. 4.



wherefore $\angle FL$ is equal to GL ; and the triangle FLG is isosceles, consequently $\angle LGF$ is equal to $\angle LFG$; but $\angle LGF$ is given because it is equal to the vertical angle HGD , which is given; wherefore $\angle GFL$ is given, and consequently the re-

maining angle GFE is given, and therefore the remaining angle EFB is given.

Therefore, if a right line, given in magnitude, be bounded by two parallel lines given in position, the angles, which that line makes with the parallels, are given in magnitude.

PROPOSITION XXXIV.

If from a given point a right line be drawn, cutting two parallel right lines given in position, that right line will cut by them in a given ratio.

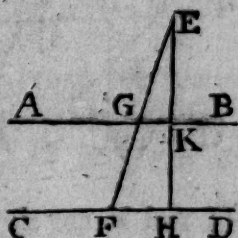
This proposition admits of two cases, because the given point may either be without or within the two parallel lines. And first let it be without them.

Let AB CD be two parallel right lines given in position, and E a given point, and from the point E let EF be drawn, cutting the two parallel lines AB CD in the points G and F ; I say the ratio of EG to GF is given. For from

Prop. 12.
 Euc. lib. 1.

the point E draw EH perpendicular to CD and

and cutting AB in K; then because the line EH is drawn from the given point E, cutting CD, given in position, at the given angle EHF, wherefore EH^b is given in position; but CD is given in position, therefore the point H is given^c; for the same reason the point K is given, and consequently^d the right lines EH KH and EK



^b Dat. 30.

^c Dat. 25.

^d Dat. 26.

are each given in magnitude, wherefore^e the ratio of EK to KH is given. But because GK is parallel to FH, as EK is to KH, so is EG to FG^f; but the ratio of EK to KH is given, wherefore the ratio of EG to GF is given.

^e Dat. 1.

^f Prop. 2.
Euc. lib. 6.

Therefore, if from a given point a right line be drawn, cutting two parallel right lines given in position, that right line will be cut by them in a given ratio; which was to be demonstrated.

CASE II.

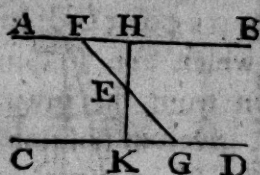
When the given point E is between the parallel lines AB and CD, from E draw EK perpendicular to CD, and which cuts AB in H^g; then because EK is drawn from the given point E, perpendicular to CD given in position, wherefore EK^h is given in position; but CD is given in position, therefore



^g Prop. 12.
Euc. lib. 1.

^h Dat. 30.

¹ Dat. 25. therefore ¹ the point K is given for the same reason the point H is given. Since the points



² Dat. 26.

¹ Dat. 1.

K E and H are all given, therefore the lines KE EH ^k are each given in magnitude, and consequently ¹ the ratio of KE to EH is given.

But because the angles at

H and K are right, and the angle FEH equal

^m Prop. 15. to GEK ^m, wherefore the triangles FEH GEK Euc. lib. 1. are equiangular, and ⁿ as KE is to EH, so is ⁿ Prop. 4. GE to EF: but the ratio of KE to EH is given, Euc. lib. 6.

^o Def. 2. therefore ^o the ratio of GE to EF is given, of this.

Therefore, if from a given point a right line be drawn, cutting two parallel right lines given in position, that right line will be cut by the parallels in a given ratio; which was to be demonstrated.

PROPOSITION XXXV.

If from a given point a right line be drawn, cutting a right line given in position, and if that right line be cut into two parts which have a given ratio to one another, and if through the point of section a right line be drawn parallel to the right line given in position, that right line is also given in position.

Let AB be a right line given in position, and C a given point, and from C let CD be drawn, cutting AB in D, and let CD be cut in E, so

I

that

that CD has to DE a given ratio, and through E let FG be drawn, parallel to AB; I say FG is given in position. For from the point C draw CH perpendicular to AB^p which cuts FG in^p Prop. 12. K; then because CH is drawn from the given^p Euc. lib. 1. point C, cutting the line AB, given in position, at a given angle, therefore^q CH is given in^q Dat. 30. position; but AB is given in position^r, therefore^r the point H is given; but C is given, wherefore^r CH is given in magnitude. Again, because EK is parallel to DH, wherefore^v as DE is to EC, so is HK to KC; but the ratio of DE to EC is given, therefore^w the ratio of HK to KC is given, and consequently^x the ratio of HC to CK is given;^x Dat. 6. but HC is given in magnitude, therefore^y KC^y Dat. 2. is given in magnitude; but it is given in position, and the extremity C is given, wherefore^z Dat. 27. the point K is given; hence FG is drawn thro' the given point K, parallel to AB which is given in position, wherefore^a FG is given in position.^a Dat. 28.



^r Hyp.
^r Dat. 25.
^r Dat. 26.

^v Prop. 2.
^v Euc. lib. 6.

^w Def. 2.
^w Dat.
^x Dat. 6.
^y Dat. 2.

Therefore, if from a given point a right line be drawn, cutting a right line given in position, and if that right line be cut into parts which have a given ratio to one another, and if through the point of section a right line be drawn parallel to the right line given in position, that right line is also given in position; which was to be demonstrated.

E

P R O.

P R O P O S I T I O N X X X V I.

If from a given point a right line be drawn, cutting a right line given in position, and to it be added another line to which the line drawn has a given ratio, and if through the extremity of that added line a right line be drawn parallel to the right line given in position, that line is also given in position.

Let AB be a right line given in position, and C a given point, and from C let CD be drawn, cutting AB in D, and to CD



let CE be added in such a manner, that CD has to CE a given ratio, through the point E let FG be drawn, parallel to AB; I say FG is given in position. For from the point C draw CH

^b Prop. 12. perpendicular to AB ^b which, produced, cuts
 Euc. lib. 1. FG in K; then because CH is drawn through
 the given point C, cutting the line AB, given

• Dat. 30. in position, at a given angle, wherefore ^c CH
 is given in position; but AB is given in position,

^d Dat. 25. therefore ^d the point H is given; but C is given,

• Dat. 26. consequently ^e CH is given in magnitude. A-
 gain, because the angles at H and K are right

^f Prop. 15. angles, and the angle DCH equal to the angle

Euc. lib. 1. KCE ^f, therefore ^g the triangles DCH KCE are

^h Cor. 2. equiangular; and ^h as DC is to CE, so is HC

Prop. 32. to CK; but the ratio of DC to CE is given,

Euc. lib. 1. ⁱ Prop. 4. wherefore
 Euc. lib. 6.

wherefore the ratio of HC to CK is given; but HC is given in magnitude, therefore¹ CK is¹ Dat. 2. given in magnitude; but CK is given in position, and the extremity C is given, therefore^{k 2} Dat. 27. the point K is given; hence the line FG is drawn through the given point K, parallel to the right line AB given in position, wherefore^{1 1} Dat. 23. FG is given in position.

Therefore, if from a given point a right line be drawn, cutting a right line given in position, and to it be added another line to which the line drawn has a given ratio, and if thro' the extremity of that added line a right line be drawn, parallel to the right line given in position, that line is also given in position; which was to be demonstrated.

P R O P O S I T I O N X X X V I I .

If a right line be bounded by two parallel right lines given in position, and cut into two parts which have a given ratio to one another, and if thro' the point of section a right line be drawn parallel to the first mentioned lines, that right line is given in position.

Let AB CD be two parallel lines given in position, and the line FG, which is bounded by them, and cut in the point E, so that the ratio of FE to EG is given, and through E let HK be drawn parallel to



E 2

AB

AB or CD; I say HK is given in position. For take any point in the line AB, as L, and from

^mProp. 11.
Euc. lib. 1.



L draw LM, perpendicular to AB^m, which cuts CD in M, and HK in L; then because LM is drawn through the given point L, in the line AB given in position,

ⁿ Dat. 29. making a given angle with it, wherefore ⁿ LM is given in position; but CD is given in position^o,

^o Hyp. therefore ^p the point M is given; but L ^p Dat. 25. is given, wherefore ^q LM is given in magnitude.

^q Dat. 26. Again, because the lines AB HK CD are all parallel to one another, and cut by the lines FG

^r Prop. 2. and LM, wherefore ^r as FE is to EG, so is LN ^r Euc. lib. 6. to NM; but the ratio of FE to EG is given,

wherefore the ratio of LN to NM is given, and

^s Dat. 6. consequently ^s the ratio of LM to NM is given;

^t Dat. 2. but LM is given in magnitude, wherefore ^t NM is given in magnitude; but NM is given in

^v Dat. 27. position, and the point M is given, wherefore ^v the point N is given; hence the line HK is drawn through the given point N, parallel to AB given

^w Dat. 28. in position, wherefore ^w AK is given in position.

Therefore, if a right line, bounded by two parallel right lines given in position, be cut into two parts which have a given ratio to one another, and if through the point of section a right line be drawn parallel to these two lines, that right line is given in position; which was to be demonstrated.

PROPOSITION XXXVIII.

If a right line be bounded by two parallel right lines, given in position, and to it be added another line which has a given ratio to it, and through the extremity of that added line a right line be drawn parallel to the first mentioned lines, that right line is given in position.

Let AB and CD be two parallel right lines given in position and cut by the line EF, to which let EG, having a given



ratio to EF, be added, and through the point G, let GK be drawn parallel to AB, or CD; I say GK is given in position. For take any point,

as L, in the line AB, and from L, draw

LM perpendicular to AB^x, which produced^x Prop. 11. cuts the line CD in M, and GK in N. Then¹ Euc. lib. 1.

because LM is drawn from the point L, cutting AB, given in position, in a given angle,

wherefore^y LM is given in position; but CD^y Dat. 29.

is given in position^z, wherefore^a the point M^a Hyp.

is given; but L is given, therefore^b LM is^b Dat. 25.

given in magnitude. Again, because HK, AB,

and CD are all parallel to one another, and cut

by the lines GF and NM, it will be^c as FE is^c Prop. 2.

to EG, so is ML to LN; but the ratio of FE¹ Euc. lib. 6.

to EG is given, therefore^d the ratio of LM^d Def. 2.

LN is given; but LM is given in magnitude, of this.

E 3

where-

^e Dat. 2. wherefore ^e LN is given in magnitude; but LN is given in position, and the point L is

^f Dat. 27.  ^f the point N is given: hence there is a line, as GK, drawn through the given point N, parallel to AB given in position, where-

^g Dat. 28. fore ^g GK is given in position.

Therefore, if a right line be bounded by two parallel lines given in position, and to it be added another line, which has a given ratio to the first mentioned line, and through the extremity of that added line a right line be drawn parallel to the lines given in position, that line is given in position; which was to be demonstrated.

PROPOSITION XXXIX.

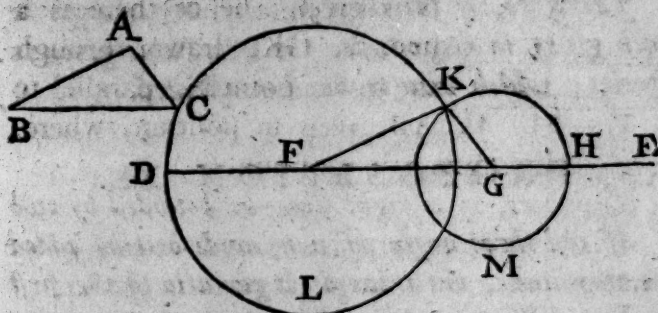
If the three sides of any triangle be each given in magnitude, the triangle is given in species.

Let the three sides AB, BC, and CA, of the triangle ABC, be each given in magnitude, I say the triangle ABC is given in species. For take any point, as D, from which let the line DE given in position be produced at pleasure. From the line DE cut off a part, as DF, equal to AB ^h, and make FG equal to BC, and GH equal to AC. About the center F, with the distance FD, describe the circle DKL, and about the center G, with the distance GH, describe the circle KHM; from the

^h Prop. 3.

Euc. lib. I.

the point of intersection K, join the lines KF, and KG. Then because DF is given in position and magnitude, and the point D is given,



therefore the point F^i is given; and since the ¹ Dat. 27. center F is given, and the radius FD is given in magnitude, wherefore ^k, the circum-^k Def. 6. ference DKL is given in position. For the ^{of this.} same reason the point G is given, and the circumference KHM is given in position, wherefore ¹ the point of intersection K is given; ¹ Dat. 25. but the point F is given, therefore FK ^m is given ^m Dat. 26. in position and magnitude; but FG is given in position, therefore ⁿ the angle KFG is given in ⁿ Lemma magnitude; for the same reason, KGF is given ^{to dat. 30.} in magnitude. Again, because the two triangles ABC, KFG have the sides AB, AC equal to KF, KG, each to each, and the base BC equal to FG ^o, wherefore ^p the angle KFG ^{Construction.} is equal to the angle ABC, and KGF equal to ^{Prop. 8.} ACB; therefore the angles ABC, ACB are ^{Euc. lib. 1.} each given in magnitude, and consequently the remaining angle BAC ^q is given in magnitude. ^{Prop. 32.} But the sides AB, BC, and AC are each given ^{Euc. lib. 1.} and ^{Dat. 4.} in

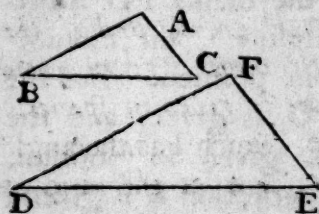
- ^r Hyp. in magnitude ^r, therefore ^r the ratios of the sides
^r Dat. 1. to each other are given; wherefore the triangle
^r Def. 3. ABC ^r is given in species.
 of this.

Therefore, if the three sides of any triangle be each given in magnitude, the triangle is given in species; which was to be demonstrated.

PROPOSITION XL.

If the three angles of a triangle be each given in magnitude, the triangle is given in species.

Let ABC be a triangle, whose three angles ABC, ACB, and BAC are each given in mag-



nitude; I say the triangle ABC is given in species. For let DE be a right line, given in magnitude and position with the line DE,

and at the points D and E construct the angles EDF and DEF, equal to ABC, and ACB,

^r Prop. 23. each to each ^r; then will the remaining angle
 Euc. lib. 1. DFE be equal to the remaining angle BAC ^r,
 Cor. 2.

^r Prop. 32. and consequently ^r the triangle ABC is similar
 Euc. lib. 1. to the triangle DFE. Again, because the

^r Prop. 4. point D is given, and DE given in position,
 Euc. lib. 6. and the angle FDE given in magnitude, be-

^r Dat. 29. ing equal to the angle ABC, wherefore ^r DF
 is given in position; for the same reason FE

^r Dat. 25. is given in position, consequently ^r the point of
 intersection F is; but the point D is given,
 there-

therefore ^b the line DF is given in magnitude ; ^b Dat. 26.
for the same reason FE is given in magnitude,
therefore the three sides of the triangle DFE
are each given in magnitude, consequently ^c Dat. 39-
the triangle DFE is given in species ; but the
triangle ABC is similar to DFE, therefore the
triangle ABC is given in species.

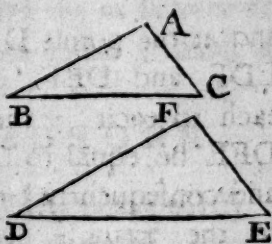
*Therefore, if the three angles of a triangle be
each given in magnitude, the triangle is given in
species ; which was to be demonstrated.*

P R O P O S I T I O N X L I .

*If a triangle has one angle given, and if the
sides about the given angle have a given ratio
to one another, the triangle is given in species.*

Let ABC be a triangle, which has the angle
ABC given in magnitude, and let the ratio of
BC to BA be likewise
given ; I say the triangle

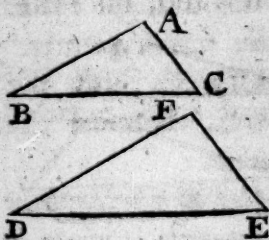
ABC is given in species.
For let DE be a right line
given in position and mag-
nitude, with the line DE,
and at the point D con-



struct the angle EDF, equal to the given an-
gle ABC ^d : and since the ratio of BC to BA is ^d Prop. 23.
given ^e, let DE have the same proportion to ^e Euc. lib. 1.
DF, then is the ratio of DE to DF given ; but ^e Hyp.
DE is given in magnitude, therefore ^f DF is ^f Dat. 2.
given in magnitude. Join the line FE. Then,
because

because the point D is given, and DE in position, and the angle EDF given in magnitude,

^g Dat. 29.



^h Dat. 27.

ⁱ Dat. 26.

^k Dat. 39. and consequently ^k the triangle FDE is given in species. Again, because the angle ABC is equal to the angle FDE, and as BC is to BA, so is DE to DF, wherefore ^l the triangle ABC is ^l Prop. 6. similar to the triangle EDF; but EDF is given in species, consequently the triangle ABC is given in species.

Euc. lib. 6.

Therefore, if one angle of a triangle be given, and if the sides about the given angle have a given ratio to one another, the triangle is given in species; which was to be demonstrated.

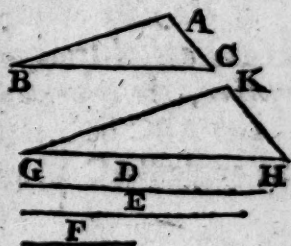
PROPOSITION XLII.

If the ratios of all the sides of a triangle, each to each, be given, the triangle is given in species.

Let ABC be a triangle, and let the ratios of AB to BC, BC to CA, and AB to AC, be all given; I say the triangle ABC is given in species. For let D be a right line given in magnitude, and since the ratio of BC to BA is

is

is given, let D have the same ratio to E, consequently the ratio of D to E is given; but D is given, therefore ^m E is given: also, since the ^m Dat. 2. ratio of BA to AC is given, let E have the same ratio to F, wherefore the ratio of E to F is given; but E is given, therefore ⁿ F is given.



ⁿ Dat. 2.

Then construct the triangle GHK, whose three sides are equal to the three given lines D, E, and F^o, any two of which are greater than the third, because they have the same proportion to one another with the three sides of the triangle ABC. Let GH be equal to D, GK equal to E, and KH equal to F; then, because the triangle GKH has all its sides given in magnitude, it is given in species ^p; but because BC is to BA, as D is to E, ^p Dat. 39. that is, as GH is to GK; and BA is to AC, as E to F, that is, as GK is to KH, by equality ^q it will be as BC is to CA, so is GH to HK, wherefore ^r the triangle ABC is similar to the triangle KGH; but the triangle KGH is given in species, consequently the triangle ABC is given in species.

^o Prop. 22.

^p Euc. lib. 1.

^q Prop. 22.

^r Euc. lib 5.

^s Prop. 5.

^t Euc. lib. 6.

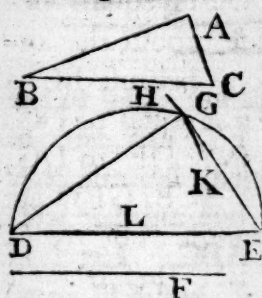
Therefore, if the ratios of all the sides of a triangle, each to each, be given, the triangle is given in species; which was to be demonstrated.

P R O-

PROPOSITION XLIII.

If the sides about one of the acute angles in a right angled triangle have a given ratio to one another, the triangle is given in species.

Let ABC be a right angled triangle, having the angle BAC a right angle, and let the ratio



^f Hypoth.

of BC to BA about the acute angle ABC be given; I say the triangle ABC is given in species. For let DE be a line given in position and magnitude^f, and because the ratio of BC to BA is given, let the ratio

of DE to F be equal to it, then is the ratio of DE to F given; but DE is given in magni-

^t Dat. 2. tude, therefore F^t is given in magnitude; but ^u Prop. 19. BC is greater than BA^u, therefore DE is greater ^v Def. 6. than FG^v. Upon DE, as a diameter, describe ^w Prop. 14. the semicircle DGE; and about the point D as ^x Def. 6. a center, with the distance F, describe NGK, which cuts DGE at G, and join the lines DG, GE: then because DE is given in position and magnitude, its half DL is given; but D is

^y Def. 6. given, therefore ^z the point L is given, and ^z consequently^y the circumference DGE is given in position. Again, because the center D is

^z Def. 6. given, and F is given in magnitude, wherefore the circumference NGK is given in position^z, where-

wherefore ^a the point of intersection G is given; ^a Dat. 25.
 but the points D and E are each given, there-
 fore ^b the lines DG and GE are given in mag- ^b Dat. 26.
 nitude, consequently ^c the triangle DGE is given ^c Dat. 39.
 in species. Again, because the angle DGE is
 a right angle ^d, therefore the angle GED is an ^d Prop. 31.
 acute angle. Hence the two triangles ABC, Euc. lib. 3.
 DGE have the angles BAC, DGE equal, be-
 ing right angles; and the sides about the an-
 gles ABC, GDE in proportion, because BC is
 to BA, as DE is to F; but F is equal to DG.
 Therefore as BC is to BA, so is DE to DG,
 and the remaining third angles ACB, GED are
 both acute, therefore ^e the triangle ABC is si- ^e Prop. 7.
 milar to the triangle DGE; but the triangle Euc. lib. 6.
 DGE is given in species, wherefore the trian-
 gle ABC is given in species.

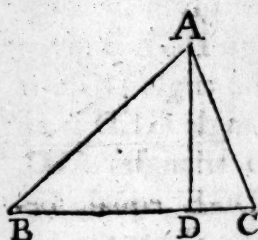
*Therefore, if the sides about one of the acute
 angles in a right angled triangle have a given
 ratio to one another, the triangle is given in
 species; which was to be demonstrated.*

PROPOSITION XLIV.

*If any angle of a triangle be given, and the
 ratio of two of its sides about another angle like-
 wise given, the triangle is given in species.*

Let ABC be a triangle, which has the angle
 ABC given, and let the ratio of AB to AC,
 about another angle BAC; be given; I say
 the triangle ABC is given in species.

This proposition admits of two cases, because the given angle ABC may be either an acute, or an obtuse angle ;



^f Prop. 12.
Euc. lib. 1.

and first let the angle ABC be acute. From the point A draw AD perpendicular to BC^f , which cuts it in D ; then, because the angle ADB is a right angle, and

^g Prop. 32. the angle ABD is given, therefore ^h the remaining angle BAD is given, and consequently ⁱ,

^j & Dat. 4. the triangle ABD is given in species, wherefore the ratio of AB to AD is given ; but the ratio

^k Hypoth. of AB to AC is given ⁱ, therefore ^k the ratio of DA to AC is given ; but the angle ADC is a

^l Dat. 43. right angle, wherefore ^l the triangle ADC is given in species, and consequently the angle ACD is given in magnitude ; but the angle

^m Hypoth. ABC is given ^m, therefore the third angle

ⁿ Prop. 32. BAC^n is given, and consequently ^o the triangle BAC is given in species.

^p Dat. 40. *Therefore, if one angle in a triangle be given, and the ratio of two of its sides about another angle likewise given, the triangle is given in species ; which was to be demonstrated.*

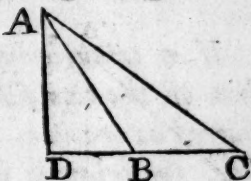
CASE II.

When the given angle ABC is obtuse, from

^p Prop. 12. the point A draw AD perpendicular to BC^p ,
Euc. lib. 1. which cuts it produced at D ; then, because
the

the angle ABC is given, the angle ABD^q is^q Prop. 13.
given; but the angle ADB is a right angle, the^q Euc. lib. 1.
remaining third angle DAB^q is given, and consequently^q & Dat. 4.

the triangle ADB is given
in species, wherefore the ra-
tio of BA to AD is given;



^q Dat. 40.

but the ratio of BA to AC is given^f, therefore^f Hypoth.

^t the ratio of DA to AC is given; but the an-^t Dat. 8.

gle ADC is a right angle, wherefore^u the tri-^u Dat. 43.

angle ADC is given in species, and conse-
quently the angle BCA is given in magnitude;

but the angle ABC is given in magnitude^w,^w Def. 3.

therefore^x the remaining angle BAC is given,^{of this.}

wherefore^y the triangle ABC is given in spe-^x Prop. 32.

cies.^{Euc. lib. 1.}

& Dat. 4.

^y Dat. 40.

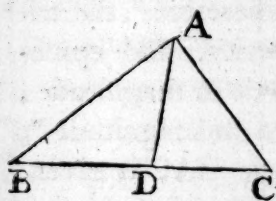
*Therefore, if one angle of a triangle be given,
and the ratio of two of the sides about another
angle likewise given, the triangle is given in spe-
cies; which was to be demonstrated.*

*N. B. This and the forty-third Proposition
might have been joined in one, for when justly
considered, they include the same hypotheses;
only by joining them in one, the proposition
would have consisted of three cases, because the
given angle may be either a right angle, an acute,
or an obtuse angle.*

PROPOSITION XLV.

If a triangle has one angle given, and if the sum of the two sides about the given angle, considered as one line, has a given ratio to the third side, the triangle is given in species.

Let BAC be a triangle, which has the angle BAC given, and let the sides BA, AC about the given angle, considered as one line, have a given ratio to the third side BC; I say the triangle BAC is given in species. For, bisect the given angle BAC



- with the right line AD^a, which cuts BC in D;
 then, because the angle BAC is bisected with
 the line AD, it is^a as BA is to AC, so is BD
 to DC; and by alternation^b, as BA is to BD,
 so is AC to CD; wherefore^c as AB is to BD,
 so is BA and AC, added together, to BD and
 DC, added together, that is, to BC; but the
 ratio of BA, AC, added together to BC, is
 given^d, wherefore the ratio of AB to BD is
 given; but the angle BAD is given, because
 it is half the given angle BAC: wherefore the
 triangle ABD, having the angle BAD given,
 and the ratio of AB to BD given, it is^e given
 in species, and consequently the angle ABC is
 given in magnitude; but the angle BAC is
 given, wherefore^f the remaining angle ACB
 is

^a Prop. 9.
Euc. lib. 1.

^b Prop. 3.
Euc. lib. 6.

^c Prop. 16.
Euc. lib. 5.

^d Prop. 12.
Euc. lib. 5.

^e Prop. 44.
Dat. 4.

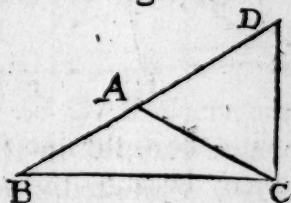
^f Prop. 32.
Euc. lib. 1.

is given, therefore ^s the triangle ABC is given ^s Dat. 40. in species.

Therefore, if one angle of a triangle be given, and if the sum of the two sides about the given angle, considered as one line, has a given ratio to the third side, the triangle is given in species; which was to be demonstrated.

This Proposition is likewise demonstrated in the following manner :

Produce the side BA, make AD equal to AC, and join DC ; then, because the angle BAC is equal to the sum of the two angles ADC and ACD ^h, and BAC is given, therefore the sum of the two angles ADC



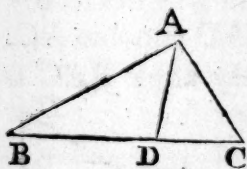
^h Prop. 32.
Euc. lib. 1.

and ACD is given; but the angle ADC is equal to ACD ⁱ, therefore each of the angles ⁱ Prop. 5. ADC and ACD are given : but the ratio of the ⁱ Euc. lib. 1. sum of BA, AC, that is, BD to BC, is given ^k, ^k Hypoth. wherefore ^l the triangle DBC is given in spe- ^l Dat. 44. cies, and consequently ^m the angle ABC is ^m Def. 3. given ; but BAC is given, therefore ⁿ the re- ⁿ of this. ⁿ Prop. 32. maining angle ACB is given, wherefore ^o the ^o Euc. lib. 1. triangle ABC is given in species ; *which was* ^o Dat. 4. *to be demonstrated.* ^o Dat. 40.

PROPOSITION XLVI.

If one angle of a triangle be given, and if the sum of the two sides about another angle, considered as one line, has a given ratio to the third side, the triangle is given in species.

Let ABC be a triangle, which has the angle ABC given, and let BA, AC , about another angle BAC , considered as one line, have a given ratio to the side BC ; I say the triangle ABC is given in species. For let



the angle BAC be bisected with the line AD ,

^p Prop. 9. which cuts BC in D ^p; then because the angle
 Euc. lib. 1. BAC is bisected, it is ^q as AB is to AC ; so is
^r Prop. 3. BD to DC ; and by alternation ^r, as AB is to
 Euc. lib. 6. BD , so is AC to DC , and therefore ^r as AB is
^r Prop. 16. BD , so is AB and AC , added together, to
 Euc. lib. 5. BD and DC added together, that is, to BC .
^r Prop. 12. Euc. lib. 5.

But the ratio of AB and AC added together to

^r Hypoth. BC is given ^r, wherefore the ratio of AB to BD
^r Hypoth. is given; but the angle ABD is given ^u, there-
^w Dat. 41. fore the triangle ABD ^w is given in species, and
^x Def. 3. consequently ^x the angle BAD is given in mag-
 of this, nitude; but $\angle BAD$ is half the angle BAC ,
 wherefore the angle BAC is given; but the an-

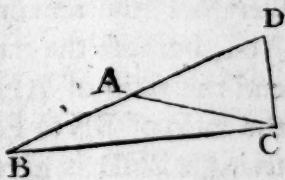
^r Prop. 32. gle ABC is given, therefore ^y the remaining
 Euc. lib. 1. angle ACB is given, and consequently ^z the
 & Dat. 4. triangle ABC is given in species.
^z Dat. 40.

Therefore,

Therefore, if one angle of a triangle be given, and if the sum of the two sides about another angle, considered as one line, has a given ratio to the third side, the triangle is given in species; which was to be demonstrated.

This Proposition is otherwise demonstrated, thus :

Produce BA, and make AD equal to AC, and join DC; then, because BD is equal to BA and AC added together, wherefore ^a the ratio of BD to BC is given; but the angle DBC is given, wherefore ^b the triangle



^a Hypoth.

^b Dat. 41.

DBC is given in species, and consequently the angle BDC is given in magnitude; but the angle BAC is double the angle BDC ^c, therefore the angle BAC is given, and therefore the remaining angle ACB is given ^d; wherefore ^e the triangle BAC is given in species; *which was to be demonstrated.*

^c Prop. 5.

and 32.

Eucl. lib. 1.

^d Prop. 32.

Eucl. lib. 1.

& Dat. 4.

^e Dat. 40.

PROPOSITION XLVII.

Any right lined figure given in species, is divisible into triangles, which are also given in species.

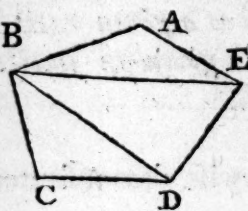
Let ABCDE be any right lined figure given in species, join the lines BE and BD; I say the triangles BAE, BED, and BCD are all

F 2

given

given in species. Because the angle BAE is
^f Hypoth. given, and the ratio of BA to AE is given ^f,

^g Dat. 41.



wherefore ^g the triangle BAE is given in species. For the same reason the triangle BCD is given in species. Again, because the angle AED is given

^h Hypoth. in magnitude ^h, and the angle AEB is given, because the triangle AEB is given in species,

ⁱ Dat. 4. therefore ⁱ the remaining angle BED is given.

^k Hypoth. Also, because the ratio of AE to ED is given ^k,

^l Dat. 8. and the ratio of AE to EB is given, therefore ^l

^m Dat. 41. the ratio of BE to ED is given, wherefore ^m the triangle BED is given in species; wherefore all the triangles ABE, BED, and BDC are each given in species.

Therefore, any right lined figure given in species, is divisible into triangles, which are each given in species; which was to be demonstrated.

PROPOSITION XLVIII.

If upon the same right line two triangles given in species be described, these triangles will have a given ratio to one another.

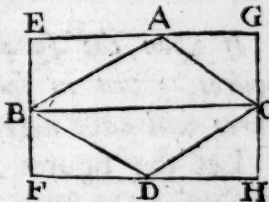
Let ABC and BCD be two triangles given in species, and described upon the same right line BC; I say the ratio of the triangle ABC to the triangle BCD is given.

For,

EUCLID'S DATA.

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For, through the two points A and D draw the right lines EG and FH parallel to BCⁿ, ⁿ Prop. 31. Euc. lib. 1. and from the points B and C draw the right lines EF and GH perpendicular to BC^o, which produced cut the lines EG and FH in the points E, G, F, and H;



• Prop. 11. Euc. lib. 1.

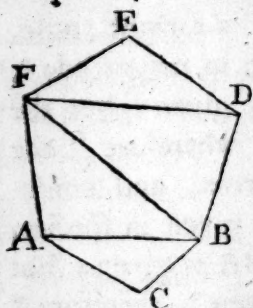
then, because the angle EBC is a right angle, and the angle ABC is given in magnitude^p, ^p Hypoth. wherefore^q the angle EBA is given; but the^q Dat. 4. angle BEA is a right angle^r, wherefore^r the^r Prop. 29. Euc. lib. 1. remaining angle EAB is given, and conse-^r Prop. 32. Euc. lib. 1. quently^t the triangle AEB is given in species, therefore the ratio of EB to BA is given; but^t Dat. 4. the ratio of BA to BC is given^u, therefore^u Dat. 40. Hypoth. the ratio of EB to BC is given. For the same^w Dat. 8. reason the ratio of BC to BF is given, where- fore^x the ratio of EB to BF is given. But as^x Dat. 8. EB is to BF, so is the parallelogram EC to the parallelogram BH^y, therefore the ratio of^y Prop. 1. Euc. lib. 6. the parallelogram EC to the parallelogram BH is given; but the parallelograms EC and BH are each double the triangles ABC, BDC^z, ^z Prop. 41. Euc. lib. 1. and parts have the same proportion to their like multiples^a, therefore the ratio of the tri-^a Prop. 15. Euc. lib. 5. angle ABC to the triangle BDC is given.

Therefore, if upon the same right line two tri- angles given in species be described, these tri- angles have a given ratio to one another; which was to be demonstrated.

PROPOSITION XLIX.

If upon the same right line two right lined figures, given in species, be described, these figures will have a given ratio to one another.

Let the figures ABC and $ABDEF$, given in species, be both described upon the same line AB ; I say these figures



^a Dat. 47.

will have a given ratio to one another. For join the lines FD , FB ; then, because the figure $ABDEF$ is given in species, therefore ^{*} the triangles EFD , FDB , and FAB are each given in

species. Since the two triangles EFD , FDB are given in species, and described upon the

^b Dat. 48. same line FD , wherefore ^b the triangle EFD has a given ratio to the triangle FDB , and con-

^c Dat. 6. sequently ^c the ratio of the trapezia $EFBD$ to the triangle FDB is given; but the ratio of

^d Dat. 48. the triangle FBD to the triangle FBA is given ^d,

^e Dat. 8. wherefore ^e the ratio of the trapezia $EFBD$ to

^f Dat. 6. the triangle FBA is given, and consequently ^f the ratio of the figure $ABDEF$ to the triangle FAB is given; but the ratio of the triangle

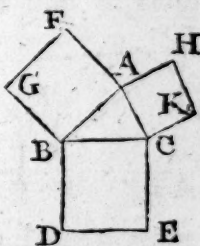
^g Dat. 48. FAB to the triangle ABC is given ^g, where-

^h Dat. 8. fore ^h the ratio of the figure $ABDEF$ to the triangle ABC is given, &c.

Therefore, if upon the same right line two right lined figures, given in species, be described, these figures will have a given ratio to one another ; which was to be demonstrated.

COROLLARY.

From this Proposition it plainly follows, that the square described upon the different sides of a triangle given in species, have each a given ratio to the triangle. For let ABC be a triangle given in species, and upon the sides let the squares BE, BF, CH be described, it is plain, that since all squares are given in species, therefore, upon the same line BC, there is described two figures ABC and BE both given in species ; wherefore, by the Proposition, the square BE has a given ratio to the triangle ABC : for the same reason, the squares BF and CH have each a given ratio to the triangle ABC.



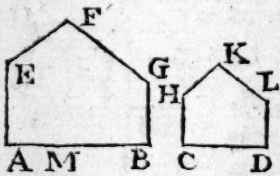
Note, This is the sixty-third Proposition in all the copies of the Data now extant ; but as Pappius makes no mention of it, it is evidently spurious, or if used by Euclid at all, it must have been used as a Corollary to some Proposition, as it evidently follows from this ; and for that reason we have inserted it here. Though I am rather

induced to think that it has been added by some other Author.

PROPOSITION L.

If two right lines have a given ratio to one another, similar figures described upon them will likewise have a given ratio to one another.

Let the two lines AB and CD have a given ratio to one another, and upon them let the similar figures EFABG, HCDLK be described; I say the figures EAB, HCD have a given ratio to one another. For find a third proportional to the two lines AB, CD, which



ⁱ Prop. 11. let be M^i ; then, because AB is to CD, as CD is to M, and the ratio of AB to CD is given ^k, therefore the ratio of CD to M is given, and consequently ^l the ratio of AB to M is given. ^m Cor. to But ^m as AB is to M, so is the figure EAB to the figure HCD, wherefore the ratio of the figure EAB to the figure HCD is given.

ⁱ Prop. 11.
Euc. lib. 1.
^k Hypoth.

^l Dat. 8.

^m Cor. to
Prop. 20.
Euc. lib 6.

Therefore, if two lines have a given ratio to one another, similar figures described upon them will likewise have a given ratio to one another; which was to be demonstrated.

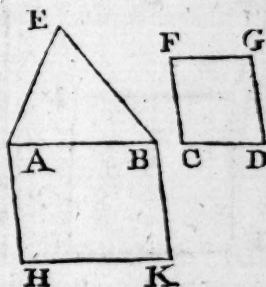
PROPOSITION LI.

If two right lines have a given ratio to one another, right lined figures given in species, described upon them, will also have a given ratio to one another.

Let AB, CD be two right lines which have a given ratio to one another, and upon them

let the figures EAB, FCD, given in species, be described; I say the ratio of EAB to FCD is given.

For upon AB let the figure AHK, similar to FCD, be describedⁿ; then, because AB has a given



ⁿ Prop. 18.
Euc. lib. 6.

ratio to CD^o, and upon them the similar figures AHK, ECD are described, wherefore^p

the ratio of AHK to ECD is given. Again, because AHK is similar to ECD, and ECD is given in species, wherefore AHK is given in species; but EAB is given in species, consequently^q the ratio of EAB to AHK is given;

but the ratio of AHK to FCD is given, wherefore^r the ratio of EAB to FCD is given.

^p Dat. 50.
^q Dat. 49.
^r Dat. 8.

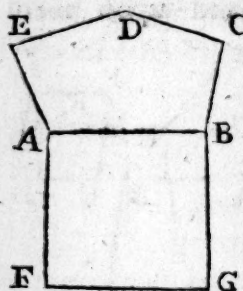
Therefore, if two right lines have a given ratio to one another, right lined figures given in species, described upon them, will also have a given ratio to one another; which was to be demonstrated.

PRO-

PROPOSITION LII.

If upon a right line given in magnitude, a right lined figure given in species be described, that figure is also given in magnitude.

Let AB be a right line given in magnitude, and upon it let the figure ABCD, given in species,



† Prop. 46.
Euc. lib. I.

† Def. 1.
and 3.
of this.

I say the figure ABCD is given in magnitude. For upon the given line AB describe the square AG[†]; then, because AG is a square described upon the given line AB[†], it is given both in species and

magnitude; but the figure ABCD is given in

‡ Hypoth. species[‡], therefore, upon the same line AB, the figures ABCD, AG are described, and

‡ Dat. 49. both given in species[‡], consequently the ratio of AG to ABCD is given; but AG is given

‡ Dat. 2. in magnitude, wherefore[‡] the figure ABCD is given in magnitude.

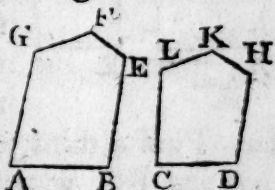
Therefore, if upon a right line given in magnitude, a right lined figure given in species be described, that figure is also given in magnitude; which was to be demonstrated.

PROPOSITION LIII.

If two figures be given in species, and if one side of the one has a given ratio to one side of the other, then will all the other sides of the one have given ratios to all the remaining sides of the other figure, each to each.

Let ABEF and CDHK be two figures given in species, and let AB have a given ratio to CD; I say AG will have a given ratio to CL, &c.

For since ABEF is given in species, the ratio of GA



to AB is given^a; but the ratio of AB to CD is given^a, wherefore^b the ratio of GA to CD is given. Again, because the figure CDH is given in species, the ratio of CD to CL is given^c; but the ratio of GA to CD is given, consequently the ratio of GA to CL^d is given. After the same manner it may be proved, that the ratios of BE to DH, of GF to LK, &c. are each given.

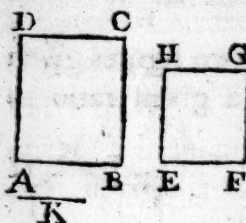
Therefore, if two figures be given in species, and if one side of the one has a given ratio to one side of the other, then will all the other sides of the one have given ratios to all the remaining sides of the other figure, each to each; which was to be demonstrated.

P R O-

PROPOSITION LIV.

If two figures given in species have a given ratio to one another, then will all the sides of the one have a given ratio to all the sides of the other, each to each.

Let ABC and EFG be two figures given in species, which have a given ratio to one another; I say their sides AB, EF, &c. will also have a given ratio to one another; for the figures ABC, EFG will either be similar, or not. And first let them be similar.

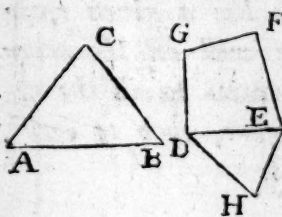


Find a third proportional to the two lines AB and EF, which let be K^d ; then, because ^{d} Prop. 11. AB is to EF as EF is to K^e , AB is to K, as ^{e} Cor. to Prop. 20. the figure ABC is to the figure EFG; but the ^{d} Prop. 11. Euc. lib. 6. ratio of the figure ABC to the figure EFG is given, therefore the ratio of AB to K is given, ^{f} Dat. 24. and consequently ^{f} the ratio of AB to EF is given, and therefore ^{g} the ratios of the remaining sides DA, BC, &c. to the remaining sides EH, FG, &c. are given. ^{g} Dat. 53.

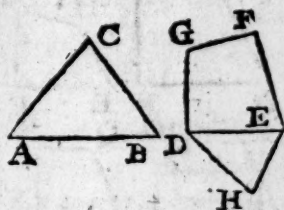
But if the figures ABC, DEF are not similar

upon the line DE, let the figure DEH be described similar to ABC ^{h} ; then because the figures ABC, DEH are similar, and ABC is given in species, therefore DEH is given in species; but DEF is given

^{h} Prop. 18.
Euc. lib. 6.



given in speciesⁱ, wherefore the two figuresⁱ Hypoth. DEF, DEH are both given in species, and described upon the same line DE, consequently^{k k} Dat. 49. the ratio of DEF to DEH is given; but the ratio of ABC to DEF is given^l, wherefore^m the ratio of ABC to DEH is given, and therefore, by the above demonstration, the ratio of AB to DE is given, andⁿ therefore the ratios of the remain-ⁿ Dat. 53. ing sides to the remaining sides are given, each to each.



^l Hypoth.
^m Dat. 8:

Therefore, if two figures be given in species, having a given ratio to one another, then will all the sides of the one have a given ratio to all the sides of the other, each to each; which was to be demonstrated.

This Proposition is demonstrated otherwise in the following manner:

The figures ABC, EFG must either be similar or not, and first let them be similar. Let KL be a line of any given magnitude, and as AB is to EF, so let KL be to MN°. Upon the lines KL and MN let the figures KLO, QMN be described similar to the figures ABC, EFG^p, and upon MN let the



^p Prop. 12.
Euc. lib. 6.

^p Prop. 18.
Euc. lib. 6.

square

¹ Prop. 46. square MS be described¹; then because AB is
 Euc. lib. 1. to EF, as KL is to MN, therefore² as the fi-
² Prop. 22. gure ABC is to the figure EFG, so is the fi-
 Euc. lib. 6. gure KLO to the figure QMN; but the ratio of

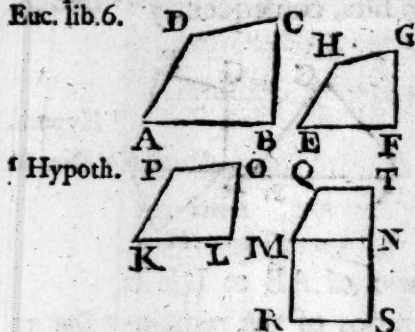


figure ABC is to the figure EFG, so is the figure KLO to the figure QMN; but the ratio of ABC to EFG is given³, therefore the ratio of KLO to QMN is given. Again, because the figures ABC, EFG are

given in species, and the figures KLO, QMN are similar to them, wherefore KLO, QMN are given in species; but the square MS is given in species, hence the figures QMN and MS are both given in species, and described

⁴ Dat. 49. upon the same line MN, consequently⁵ the ratio of QMN to MS is given; but the ratio

⁶ Dat. 8. of KLO to QMN is given, wherefore⁷ the

⁸ Dat. 52. ratio of KLO to MS is given; but KLO⁹ is given in magnitude, because it is given in species, and described upon KL, which is gi-

¹⁰ Dat. 2. ven in magnitude, therefore¹¹ the square MS is given in magnitude, and therefore its side MN is given in magnitude, wherefore the ratio of

¹² Dat. 1. KL to MM is given¹³; but KL is to MN, as AB is to EF, therefore the ratio of AB to EF is given; and since the figures ABC, EFG are given in species, and one side in the one has a given ratio to one side in the other, the ratios of their remaining sides to one another

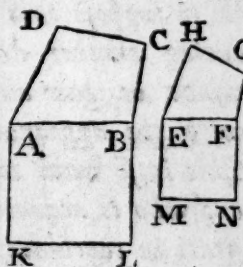
¹⁴ Dat. 53. are also given, each to each¹⁵.

But

But if the figures ABC, EFG are not similar, then may their sides, by the same construction and demonstration with the above, be proved to have a given ratio to one another, or by the following demonstration.

Let ABC, EFG be given in species, and have a given ratio to one another; I say the ratio of AB to EF, &c. is given. For upon

AB, EF let the two squares AL, EN be described^a; then because AL and EN are squares, they are given in species^b, wherefore the two figures ABC and AL are both given in species,



^a Prop. 46. Euc. lib. 1.

^b Def. 3. of this.

and described upon the same line AB, therefore^c the ratio of ABC to AL is given; but^c Dat. 49. the ratio of ABC to EFG is given, wherefore the ratio of AL to EFG^d is given. Again,^d Dat. 8. because the figures EFG and EN, given in species, are described upon the same line EF, therefore^e the ratio of EFG to EN is given; ^e Dat. 49. but the ratio of AL to EFG is given, wherefore the ratio of AL to EN^f is given; but^f Dat. 8. AL is similar to EN, therefore, by either of the preceding demonstrations, the ratio of AB to EF is given, and consequently^g the ratios^g Dat. 53. of the remaining sides of the one to those of the other are given, each to each.

Therefore,

EUCLID'S DATA.

Therefore, if two right lined figures be given in species, &c. which was to be demonstrated.

Note, This last demonstration constitutes the seventy-seventh Data in Gregory, Hardy, Serle, &c. but as it is evidently no more than a different demonstration of the second part of this Proposition, it must be spurious. For it is impossible to suppose that Euclid, for the sake of introducing another demonstration of any proposition which he has already proved, would insert it with the appearance of a different proposition. This is so much contrary to his method, that any one who is acquainted with his works cannot hesitate in concluding, that the demonstration is not his, otherwise, agreeable to his constant practice, he would have introduced it in the same manner we have done. This plainly accounts for one of the supernumerary propositions that are now found in all the copies of the Data, which are very different from what they were in Pappus's time; for in his Preface to his Seventh Book, he says there were only ninety, and in all the copies now extant there are ninety-five. The others we shall regularly account for in their proper places.

P R O-

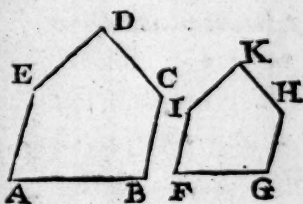
PROPOSITION LV.

If any right lined figure be given both in species and magnitude, all its sides are given in magnitude.

Let ABCD be a figure given in species and magnitude; I say its sides AB, BC, &c. are each given in magnitude.

For let FG be a right line given in position and magnitude; upon FG de-

scribe the figure FGHK,



similar to the figure ABCDE^h; then because the figure ABCD is^h Prop. 18. given in speciesⁱ, and the figure FGHK is si-^h Euc. lib. 6. Hypoth. milar to it, therefore the figure FGHK is given in species, and it is described upon the right line FG, which is given in magnitude, therefore^k the figure FGH is given in mag-^k Dat. 52. nitude; but ABCD is given in magnitude, wherefore^l the ratio of ABCD to FGHK is^l Dat. 1. given; but they are both given in species, therefore^m the ratio of AB to FG is given;^m Dat. 54. but FG is given in magnitude, thereforeⁿ ABⁿ Dat. 2. is given in magnitude; but the ratio of AB to BC is given, and AB is given in magnitude, wherefore BC is given in magnitude^p: after^p Dat. 2. the same manner all the other sides may be proved to be given in magnitude.

Therefore, if any right lined figure be given both in species and magnitude, all its sides are given in magnitude; which was to be demonstrated.

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This

This Proposition is otherwise demonstrated in the following manner :

^q Prop. 46. Upon AB describe the square AG^q; then, Euc. lib. 1. because AG is a square, it is given in species ;

but ABCD is given in species^r, and they are both described upon the same line AB, wherefore^r the ratio of ABCD to the square AG is given ; but the figure ABCD

is given in magnitude^t, wherefore^u the square AG

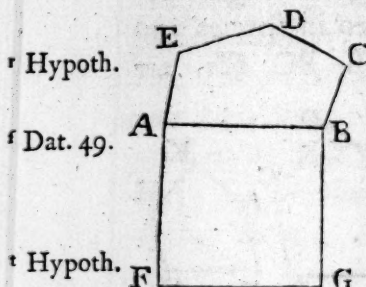
is given in magnitude ; but AG is a square described upon the side AB, wherefore the square of the side AB is given in magnitude, and consequently the line AB is given in magnitude ; but because the figure ABCD is given

in species^w, wherefore the ratio of AB to BC is given^x ; but AB is given in magnitude, therefore^y BC is given in magnitude. After the same manner it may be demonstrated, that all the other sides CD, DE, &c. are each given in magnitude.

Therefore, if the sides of any figure, &c. which was to be demonstrated.

PROPOSITION LVI.

If two equiangular parallelograms have a given ratio to one another, one side in one of them will have the same given ratio to that right line,



^r Hypoth.

^r Dat. 49.

^t Hypoth.

^u Dat. 2.

^w Hypoth.

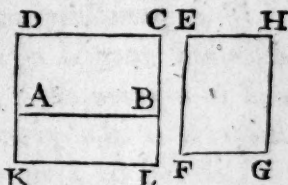
^x Def. 3.

of this.

^y Dat. 2.

line, which is a fourth proportional to the other side of that first mentioned parallelogram; and the two sides of the other parallelogram.

Let AC and EG be two equiangular parallelograms, that have a given ratio to one another; I say that AD, which is one side of the parallelogram AC, will have the same given ratio to that line which is the



fourth proportional to AB, EF and FG, which the parallelogram AC has to the parallelogram EG. For produce DA, and as AB is to EF, so let FG be to AK^z, and complete the parallelogram AL. Then because AB is to EF, as FG, or EH is to AK, and the angle HEF is equal to the angle BAK^a, wherefore^b the parallelogram AL is equal to the parallelogram EG. But AC has a given ratio to EG, and AC has the same ratio to AL that it has to EG, therefore the ratio of AC to AL is given; but as AC is to AL^c, so is DA to AK, therefore the side AD has the same given ratio to AK, that the parallelogram AC has to the parallelogram EG; but AK is the fourth proportional to AB, EF, and FG.

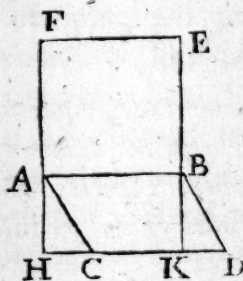
Therefore, if two equiangular parallelograms have a given ratio to one another, one side in one of them will have the same given ratio to that right line, which is a fourth proportional to the other side of that first mentioned parallelogram,

and the two sides of the other parallelogram ;
which was to be demonstrated.

PROPOSITION LVII.

If a parallelogram, given in magnitude, be described upon a right line given in magnitude, and in a given angle ; the other side of the parallelogram is also given in magnitude.

Let AB be a right line given in magnitude, upon which is described the parallelogram AD



^d Prop. 46.
Euc. lib. 1.

given in magnitude, and having the angle BAC given in magnitude ; I say the other side AC is also given in magnitude. For upon AB describe the square AE^d, produce FA till it cut CD produced in H, and compleat

the parallelogram AK ; then, because AB is

^e Dat. 52. given in magnitude, therefore^e the square AE is given in magnitude ; but AK is given, be-

^f Prop. 35. cause it is equal to AD^f, wherefore^g the ratio of AE to AK is given ; but as AE is to AK,

^h Dat. 1. so is AF to AH^h, wherefore the ratio of FA, ^a Prop. 1. Euc. lib. 6. that is of AB to AH, is given. Again, be-

cause the angle BAH is a right angle, and the ⁱ Dat. 4. angle BAC is given, thereforeⁱ the angle HAC is given ; but the angle AHC is a right angle,

^k Prop. 32. wherefore^k the remaining angle ACH is given, & Dat. 4. consequently^l the triangle AHC is given in

^l Dat. 40.

species,

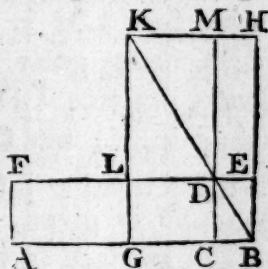
species, and hence the ratio of AH to AC is given; but it has been demonstrated, that the ratio of AH to AB is given, therefore ^m the ^m Dat. 8. ratio of AB to AC is given; but AB ⁿ is ^a Hypoth. given in magnitude, wherefore ^o AC is given ^o Dat. 2. in magnitude.

Therefore, if a parallelogram, given in magnitude, be described upon a right line given in magnitude, and in a given angle; the other side of the parallelogram is also given in magnitude; which was to be demonstrated.

PROPOSITION LVIII.

If to a right line given in magnitude, a parallelogram given in magnitude be applied, wanting in figure by a parallelogram given in species, the sides of that last mentioned parallelogram are also each given in magnitude.

Let AB be a right line given in magnitude, to which the parallelogram AD, given in magnitude, is applied, wanting in figure by the parallelogram CE, which is given in species; I say the sides CB, CD of that parallelogram are also given in magnitude. For bisect AB

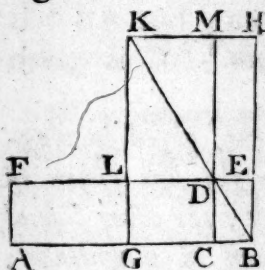


at G^p, and upon GB describe the parallelogram ^p Pron. 10. GH similar to CE ^q, and produce CD to M; ^q Prop. 18. then because the parallelogram CE is similar ^q Prop. 18. to

to the parallelogram GH, therefore the parallelogram CE stands about the diameter of GH^r.
^r Prop. 26. Euc. lib. 6.

Again, because AB is given in magnitude,

therefore its half, GB, is given; but GH is given in species, because it is similar to CE, which is given in species^t, wherefore GH^t is given in magnitude.



^r Hyp.

^t Dat. 52.

Also, because GD is equal to DH^u, wherefore GE is equal to CH; but GE is equal to AL^w, wherefore AL is equal to CH, and consequently the parallelogram AD is equal to the gnomon GEM; but AD

is given in magnitude^x, therefore the gnomon GEM is given in magnitude: before it was demonstrated that the whole, GH, was given in magnitude, and now the gnomon GEM is

given, therefore^y the remaining parallelogram LM is given in magnitude; but because LM

is similar to CE^z, and CE is given in species, wherefore LM is given in species; wherefore LM being given both in species and magni-

tude, its side LD, that is GC^a, is given in magnitude; but GB is given, therefore the remainder, CB, is given^b; but the ratio of CB

to CD is given, and CB is given, therefore

CD^c is given in magnitude.

Therefore, if to a right line given in magnitude, a parallelogram given in magnitude be applied, wanting in figure by a parallelogram given

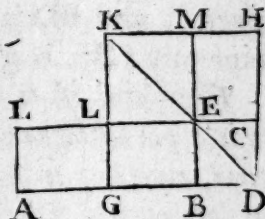
in

in species, the sides of that last mentioned parallelogram are also each given in magnitude; which was to be demonstrated.

PROPOSITION LIX.

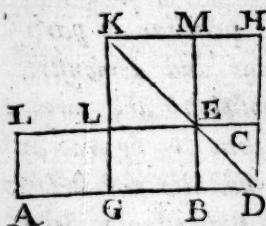
If to a right line given in magnitude a parallelogram given in magnitude be applied, exceeding in figure by a parallelogram given in species, the sides of that last mentioned parallelogram are each given in magnitude.

Let AB be a right line given in magnitude, to which the parallelogram AC, given in magnitude, is applied, exceeding in figure by the parallelogram BC given in species; I say the sides BC and BE of the parallelogram BC are each given in magnitude.



For bisect AB at G^d, and upon GD^d Prop 10. describe the parallelogram GH, similar to BC^e; Euc. lib. 1. then will BC stand about the diameter of the parallelogram GH^f, and LM is about the same diameter, therefore LM will be similar to BC^g, and consequently it is given in species; but because AB is given in magnitude, its half, GB or LE, which is equal to it, is given in magnitude; hence the figure LM, given in species, is described upon the given line LE, wherefore LM^h is given in magnitude. Again, because GE is equal to EHⁱ,¹ Prop. 43. and Euc. lib. 1.

^k Prop. 36. and AL equal to GE ^k, therefore AL is equal to EH, and consequently the whole parallelo-



gram AC is equal to the gnomon GCM; but AC is given in magnitude, wherefore the gnomon GCM is given in magnitude; but LM is given,

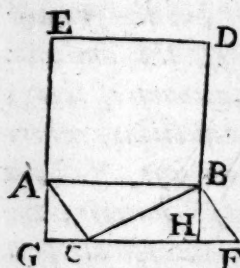
- ¹ Dat. 8. wherefore ¹ the whole parallelogram GH is given in magnitude; but GH is given in species, wherefore ^m GD is given in magnitude;
^m Dat. 55. but GB is given, therefore ⁿ the remainder, BD, is given; but the ratio of BD to BE is given ^o, and BD is given in magnitude, consequently ^p BE is given in magnitude.
^o Hyp.
^p Dat. 2.

Therefore, if to a right line given in magnitude a parallelogram given in magnitude be applied, exceeding in figure by a parallelogram given in species, the sides of that last mentioned parallelogram are each given in magnitude; which was to be demonstrated.

N. B. The enuntiations of these three last Propositions, in all the copies that have fallen into our hands, are evidently corrupted; because, in place of a parallelogram or triangle given in magnitude, they all mention a given space, at the same time the constructions and demonstrations, used by them, are the same with those above: from which it is plain, that since the thirty-fifth, sixth, and forty third Propositions of the
First

First Book, as well as the twenty-fourth and twenty-sixth Propositions of the Sixth Book, demonstrate properties which are peculiar to parallelograms, and the constructions and demonstrations of these three last Propositions are formed from them, it plainly follows that the figures applied to the given right line, and also the defects and excess in the two last, must necessarily belong to parallelograms or triangles, otherwise the demonstrations, and of consequence the propositions themselves, are imperfect, and not demonstrated. As this loose and ambiguous manner of expression by no means corresponds to that accuracy and precision by which Euclid expresses himself in his other works, we may therefore, with justice to such a great GEOMETRICIAN, conclude, that the error in these Propositions, as well as some others in the Data, are derived from the Arabians, who copied him, and from whom we had not only such works of his that are now extant, but the greatest part of the other fragments of the ancients. This corresponds with Pappus's account of them; for in his seventh book of his collections, where he speaks of Euclid's Data, he says, " That
" Euclid's Data contains ninety Theorems; the
" twenty-three first Propositions demonstrate pro-
" perties of magnitudes in general; the twenty-
" fourth is concerning three right lines in continued
" proportion; the fourteen following, and which
" go to the thirty eighth Proposition, treat of
" the properties of right lines given in position;
" the

“ the next ten, which terminate at the forty-
 “ eighth, are concerning triangles given in spe-
 “ cies, without any regard to their position; the
 “ next seven, that is to the fifty-fifth, are con-
 “ cerning right lined figures in general given in
 “ species, without any regard to their position;
 “ the next six, and which consequently include the
 “ 56th, 57th, 58th, 59th, 60th, and 61st Pro-
 “ positions, treat of the properties of parallelo-
 “ grams and spaces given in species, applied to
 “ given right lines:” by which it is obvious,
 that the spaces meant here by Pappus are paral-
 lelogrammic spaces, and that of consequence, in the
 copies of Euclid's Data, which were perused by
 Pappus, these Propositions were limited to pa-
 rallelograms only, or at least to triangular and
 parallelogrammic spaces; and to no other figure
 can these Propositions be applied. That they
 may be applied to triangles is evident: for



upon the given line AB let
 the triangle ABC, given in
 magnitude, having the angle
 BAC given in magnitude, be
 described; I say the other side,
 AC, is given in magnitude.
 For compleat the parallelogram
 AF which has AB, AC for its two adjacent sides;
 then, by the same construction and demonstration
 with that fifty-seventh Proposition, the line AC
 may be proved to be given in magnitude.

Again, to the given line AB let the triangle ACD given in magnitude, be applied, the first, wanting in figure by the triangle DCB, given in species, and the other, exceeding in figure by the triangle DCB, given in species; I say, in both these cases, the lines DC, DB, as in fig. first, and the lines BC, BD, in fig. second, are given in magnitude. For com-

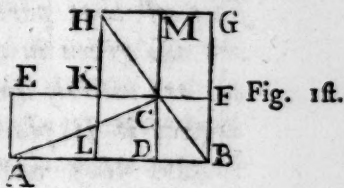


Fig. 1st.

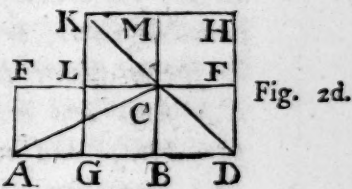


Fig. 2d.

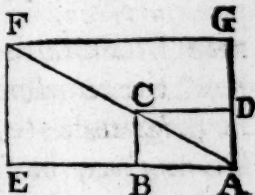
pleat the parallelograms ED, DF, in fig. first, and AF, BF, in fig. second, then, by the same construction and demonstration as before, it may be proved that the right lines DC, DB, in fig. first, and BC, BD, in fig. second, are given in magnitude. All this is evident from inspection of the figures. Hence it is plain these Propositions might have been enuntiated after this manner: If a parallelogram^z, or a triangle, given in magnitude, be^z Dat. 57. described upon a given right line, and in an angle given in magnitude; the other side of that parallelogram, or triangle, about the given angle, is given in magnitude. And the 58th and 59th might have been expressed thus: If to a right line given in magnitude a parallelogram, or a triangle, given in magnitude, be applied, wanting in figure, or exceeding in figure, by a parallelogram,

gram, or a triangle, given in species; the two sides of that parallelogram, or of the triangle, are also given in magnitude. As these Propositions are intirely limited to parallelograms and triangles, it is plain that in the genuine copies of Euclid they were enuntiated in that manner; and by an injudicious commentator, their enuntiations, for brevity's sake, have been altered from parallelograms, or triangles, given in magnitude, to spaces given in magnitude; by which the true sense and meaning of these Propositions are perverted, and rendered false. It is therefore surprising that neither Hardie, Gregory, Searl, and others, who have favoured the publick with editions of this valuable work, have corrected the corrupted enuntiations of these three Propositions, and several others; which we shall take notice of in their order. Dr. Gregory tells us in his Preface to Euclid's works, that notwithstanding he thinks that the order of the propositions in the Data have, since Pappus's time, been changed, yet he choosed, for the sake of citations, to follow the order of the other copies. Perhaps it was the same reason that induced him to retain the enuntiations of such of them as were spurious.

PROPOSITION LX.

*If to, or from a parallelogram, given in magnitude and species, a gnomon of a given magnitude be added, or taken away (so as to make the whole, or the remaining parallelogram stand about the same diameter with the given parallelogram *) the sides of that gnomon are also given in magnitude.*

Let BD be any parallelogram, given in species and magnitude, and first to it let the gnomon BEFGD given in magnitude be added ; I say the lines BE and DG are each given in magnitude.



Because the parallelogram BD is given in magnitude, and the gnomon BEFGD is given in magnitude, therefore^a the whole parallelogram EG is given^a Dat. 3. in magnitude ; but because BD is similar to EG^b, ^b Prop. 24. and BD is given in species, therefore EG is ^{Euc. lib. 6.} given in species ; wherefore EG is both given in species and magnitude, consequently^c the^c Dat. 55.

* This circumstance within the parenthesis is not inserted in the enuntiation of the Proposition in any of the other copies ; but it is plain, from the citation of the 24th Proposition of the Sixth Book, that it is supposed in the enuntiation, otherwise that step, and consequently the whole demonstration, is false.

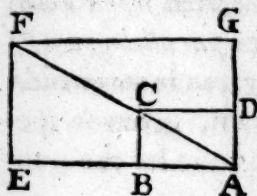
fides

sides AE, AG are given in magnitude; but

^e Dat. 55. AB, AD are each given in magnitude^e, there-

^d Dat. 4. fore the remaining lines BE, DG^d are also given in magnitude.

Again, let the parallelogram EG be given in magnitude and species, and from it let the gnomon BEFGD, given in mag-



nitude, be taken away; I say the sides BE, DG are also

each given in magnitude. For since EG is given in

magnitude, and the gnomon BEFGD is given in magnitude, there-

^e Dat. 4. fore^e the remaining parallelogram BD is given

^f Prop. 24. in magnitude; but BD is similar to EG^f, and

Euc. lib. 6. EG is given in species, therefore BD is given

in species; wherefore BD is both given in species and in magnitude, consequently^g its sides

^g Dat. 55. AB, AD are each given in magnitude; but the sides AE, AG are given, wherefore the remaining lines^h BE, DG are each given in mag-

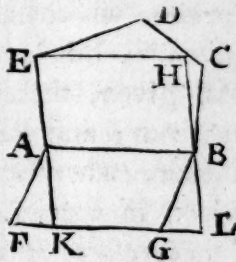
nitude.

Therefore, if to, or from a parallelogram, given in magnitude and species, a gnomon of a given magnitude be added or taken away (so as to make the whole, or remaining parallelogram stand about the same diameter with the given parallelogram) the sides of that gnomon are also given in magnitude; which was to be demonstrated.

PROPOSITION LXI.

If upon one side of a right lined figure, given in species, a parallelogram be described, having a given angle, and if the figure has a given ratio to the parallelogram, the parallelogram is given in species.

Let ABCD be a figure given in species, and upon the side AB let the parallelogram AG be described, which has the angle BAF given in magnitude, and let ABCD have a given ratio to AG; I say the parallelogram AG is given in species. For from the point B draw BH parallel to AEⁱ, and compleat the parallelogram AH;ⁱ produce the lines EA, HB, and let them cut FG produced at K and L.



Then because ABCD is given in species, the angle EAB is given in magnitude, and the ratio of AB to AE is given^k, wherefore^k Def. 3. the parallelogram AH is given in species, and of this. consequently^l the ratio of ABCD to AH is^l Dat. 49. given; but the ratio of ABCD to AG^m is gi-^m Hypoth. ven, whereforeⁿ the ratio of AH to AG isⁿ Dat. 8. given; but the parallelogram AG is equal to the parallelogram AL^o, wherefore the ratio of^o Prop. 35. AH to AL is given; but AH is to AL, as^o Euc. lib. 1. EA is to AK^p, wherefore the ratio of EA to AK^p Prop. 35. is^p Euc. lib. 1.

is given ; but the ratio of EA to AB is given,
 a Dat. 8. wherefore ^a the ratio of AK to AB is given.

Again, because the angle
 BAF is given, and BAK is
 given, wherefore ^r the angle
 KAF is given ; but since
 BAF is given, the angle
 AFK is given ^f, therefore
 the remaining FAK is given ^t,

r Dat. 4.

f Prop. 29.

Euc. lib. 1.

& Dat. 4.

t Prop. 32.

Euc. lib. 1.

& Dat. 4.

u Dat. 40.

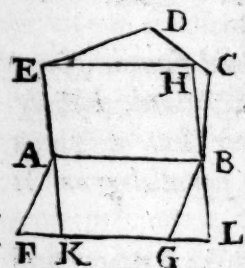
w Def. 3.

of this.

x Dat. 8.

y Def. 3.

of this.



consequently ^u the triangle AKF is given in
 species, wherefore the ratio of FA to AK is
 given ^w ; but before we had the ratio of AK to
 AB given, therefore ^x the ratio of AF to AB
 is given ; and the angle BAF is given in mag-
 nitude, wherefore ^y the parallelogram AG is
 given in species.

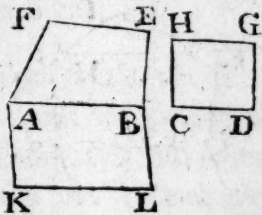
*Therefore, if upon one side of a right lined
 figure, given in species, a parallelogram be de-
 scribed, having a given angle, and if the figure
 has a given ratio to the parallelogram, the paral-
 lelogram is given in species ; which was to be
 demonstrated.*

PROPOSITION LXII.

*If two right lines have a given ratio to one
 another, and if a right lined figure, given in spe-
 cies, be described upon one of them, and a paral-
 lelogram upon the other, having one of its angles
 given, and if the figure has a given ratio to the
 parallelogram, the parallelogram is given in spe-
 cie.*

Let

Let AB, CD be two right lines that have a given ratio to one another, and let the figure ABEF, given in species, be described upon AB, and the parallelogram DH described upon CD, having the angle DCH given, and let the figure ABE



have a given ratio to DH; I say DH is given in species. For upon AB describe the parallelogram AL, similar to DH²; then because² Prop. 19. Euc. lib. 6. AB has a given ratio to CD, and upon them similar figures are described, wherefore² DH² Dat. 50. has a given ratio to AL; but ABE has a given ratio to DH^b, wherefore^c ABE has a given^b Hypoth. ratio to AL; but the angle BAK is given, because it is equal to GHC, which is given: hence, upon AB, the figure ABE is described, given in species, and the parallelogram AL, having a given angle in it; and ABE has a given ratio to the parallelogram AL, wherefore^d Dat. 61. AL is given in species; but AL is similar to DH, wherefore the parallelogram DH is given in species.

Therefore, if two right lines have a given ratio to one another, and if a right lined figure given in species be described upon one of them, and a parallelogram upon the other, having one of its angles given, and if the figure have a given ratio to the parallelogram, the parallelogram is given in specie; which was to be demonstrated.

H

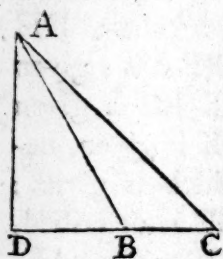
P R O-

PROPOSITION LXIII.

which is the 64th in other copies.

If the obtuse angle in any obtuse angle triangle be given, the excess by which the square described upon the side subtending the obtuse angle exceeds the sum of the two squares described upon the two sides, containing the obtuse angle, has a given ratio to the triangle.

Let ABC be an obtuse angled triangle, which has the obtuse angle ABC given; I say



the excess, by which the square described upon AC exceeds the sum of the two squares described upon AB and BC, has a given ratio to the triangle ABC. For produce BC, and from A let fall AD perpendicular to

^e Prop. 12. BC^e, which cuts it produced in D.

Euc. lib. 1.

Then because the angle ABC is given, there-

^f Prop. 13. fore^f the angle ABD is given; and the angle

Euc. lib. 1

& Dat. 4. ADB is a right angle, therefore the remaining

^g Prop. 32. angle DAB^g is given; wherefore^h the triangle

Euc. lib. 1.

& Dat. 4. ADB is given in species, and consequentlyⁱ the

^h Dat. 40. ratio of AD to DB is given; but as AD is to

ⁱ Def. 3. DB, so is the rectangle contained between AD and BC to the rectangle contained between BD

and BC to the rectangle contained between BD

^k Prop. 1. and BC^k, therefore the ratio of the rectangle

Euc. lib. 6.

contained between AD and BC, to that between BD and BC, is given, and consequently the

ratio

ratio of the rectangle contained between AD and BC, to twice that which is contained between DB and BC, is given; but the rectangle contained between AD and BC is double the triangle ABC¹, and twice the rectangle contained between DB and BC, is the excess by which the square of AC exceeds the sum of the two squares described upon AB and BC^m; therefore the ratio of the excess, by which the square of AC exceeds the sum of the two squares described upon the sides AB and BC, to twice the triangle ABC, is given, and consequently the ratio of that excess to the triangle ABC itself is given.

¹Prop. 41.
Euc. lib. 1.

^mProp. 12.
Euc. lib. 2.

Therefore, if the obtuse angle in any obtuse angle triangle be given, the excess by which the square described upon the side subtending the obtuse angle exceeds the sum of the two squares described upon the two sides, containing the obtuse angle, has a given ratio to the triangle; which was to be demonstrated.

PROPOSITION LXIV.

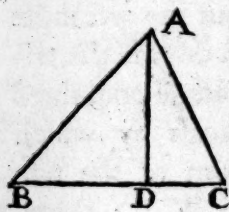
in other copies the 65th.

If any acute angled triangle has one of its acute angles given, the excess by which the sum of the two squares, described upon the two sides containing the given angle, exceeds the square described upon the side subtending the given angle, has a given ratio to the triangle.

H 2

Let

Let ABC be an acute angled triangle, which has the acute angle ABC given; I say the excess by which the sum of the



two squares, described upon AB and BC , exceeds the square of AC , has a given ratio to the triangle ABC .

For, from the point A , draw

^a Prop. 12. AD perpendicular to BC ^a; then, because the
 Euc. lib. 1. angle ABC is given, and the angle ADB is a right angle, therefore the remaining angle BAD

^b Prop. 32. is given^b, and consequently the triangle ABD
 Euc. lib. 1. ^c is given in species, wherefore^c the ratio of
 & Dat. 4.

^d Dat. 40. AD to BD is given; but as AD is to BD , so
^e Def. 3. is the rectangle contained between AD and BC .
 of this.

to the rectangle contained between BD and
^f Prop. 1. BC ^f, wherefore the ratio of the rectangle con-
 Euc. lib. 6. tained between AD and BC , to that contained
 between BD and BC is given, and consequently
 the ratio of the rectangle contained between
 AD and BC , to twice the rectangle contained
 between BD and BC , is given; but the rectan-
 gle contained between AD and BC is equal to

^g Prop. 41. twice the triangle ABC ^g, and twice the rectan-
 Euc. lib. 1. gle contained between BD and BC is, the ex-
 cess by which the sum of the two squares, de-
 scribed upon AB and BC , exceeds the square

^h Prop. 13. of AC ^h; wherefore the ratio of the excess, by
 Euc. lib. 2. which the two squares described upon AB and
 BC , exceeds the square of AC , to twice the
 triangle ABC , is given, and consequently the

ratio

ratio of that excess, the triangle ABC itself, is given.

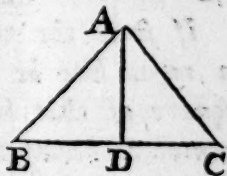
Therefore, if any acute angled triangle has one of its acute angles given, the excess by which the sum of the two squares, described upon the two sides containing the given angle, exceeds the square described upon the side subtending the given angle, has a given ratio to the triangle; which was to be demonstrated.

PROPOSITION LXV.

or 66th in other copies.

If any triangle has any one of its angles given, the rectangle contained between the two sides, about the given angle, has a given ratio to the triangle.

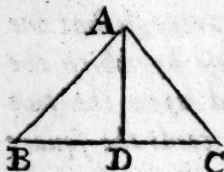
Let ABC be any triangle, which has one of its angles, as ABC, given; I say the rectangle contained between the two sides AB and BC, about the given angle ABC, has a given ratio to the triangle ABC.



For, from the point A, let

fall the line AD perpendicular to BC^u; then,^u Prop. 12. because the angle ABC is given^w, and the an-^w Euc. lib. 1. gle ADB is a right angle, wherefore the re-^w Hypoth. maining angle BAD^x is given, and therefore^y Prop. 32. the triangle ABD is given in species, and con-^z Euc. lib. 1. sequently^z the ratio of AB to AD is given; ^{& Dat. 4.} but as AB is to AD, so is the rectangle con-^z Dat. 40. tained^z Def. 3. of this.

tained between AB and BC, to the rectangle
 * Prop. 1. contained between AD and BC^a; therefore the
 Euc. lib. 6.



ratio of the rectangle contained between AB and BC, to the rectangle contained between AD and BC, is given; but the rectangle contained
 between AD and BC is double the triangle ABC^b,
 therefore the ratio of the rectangle contained between AB and BC, to twice the triangle ABC, is given, and consequently the ratio of the rectangle contained between AB and BC, to the triangle ABC itself, is given.

Therefore, if any triangle has one of its angles given, the rectangle contained between the two sides, about the given angle, has a given ratio to the triangle; which was to be demonstrated.

LEMMA.

If from the vertex of an isoscelous triangle a right line be drawn, cutting the base, the square of that line, together with the rectangle contained between the segments of the base cut off by it, will be equal to the square of one of the equal sides of the triangle.

Let ABC be an isoscelous triangle, which has the side AB equal to the side AC, and from the vertex A let AD be drawn, cutting the base BC in D; I say the square of AD, together with the rectangle contained between
 BD

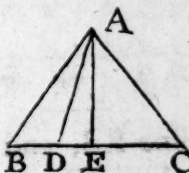
BD and DC, will be equal to the square of AB. For AD must either bisect the base or not; and first let it bisect the base. Then, because the two triangles ADB, ADC have all their three sides equal to one another, each to each^c, the angle ADB is equal to the angle ADC, and therefore right angles; wherefore^d the square of AB is equal to the two squares described upon the two sides AD and DB; but the square of BD is equal to the rectangle contained between BD and DC, because BD is equal to DC, therefore the square of AD, together with the rectangle contained between BD and DC, are equal to the square of AB.



^c Prop. 8.
Euc. lib. 1.

^d Prop. 47.
Euc. lib. 1.

But if AD does not bisect the base BC, let the base be bisected at E, join EA, then will each of the angles AEB, AEC be right angles, as before: then because the line BC is equally cut at E, and unequally cut at D, the rectangle contained between BD and DC, together with the square of DE, are equal to the square of BE^e; add the square described upon AE to both, then will the rectangle contained between BD and DC, together with the two squares described upon DE and EA, be equal to the two squares described upon BE and EA. But the square of AB is equal to the two squares of BE and



^e Prop. 5.
Euc. lib. 2.

[†]Prop. 47. EA^2 , and, for the same reason, the square of
 Enc. lib. 1. AD is equal to the two squares of DE and
 EA , therefore the rectangle contained between
 BD and DC , together with the square of AD ,
 will be equal to the square described upon AB .
Therefore, &c. which was to be demonstrated.

PROBLEM.

*From a given point, out of an infinite right
 line, given in position, to draw a line making an
 angle with the given line, equal to a right lined
 angle given.*

Let AB be a given right line C , a point out
 of it, D , a given right lined angle, it is requir-
 ed, from the point C , to draw a
 line, making an angle with the
 line AB , equal to the given an-
 gle D ; take any point in the
 line AB , as E , and with the



line EB , and at the point E , construct the an-
[‡]Prop. 23. gle BEF , equal to the angle D^e ; through the
 Euc. lib. 1. point C , draw the line CG parallel to EF^h ;
[†]Prop. 31. Euc. lib. 1. then, because CG is parallel to FE , and cut
 by the line AB , the outward angle FEB is
[†]Prop. 29. equal to the inward opposite angle CGE^i ; but
 Euc. lib. 1. the angle FEB is equal to the given angle D ,
 therefore the angle CGE is equal to the given
 angle D ; hence the line CG is drawn from the
 given

given point C, making the angle CGE equal to the given angle D; *which was to be done.*

PROPOSITION LXVI.

or the 67th in the other copies.

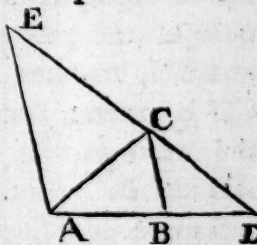
If any right lined triangle have one of its angles given, then will the excess, by which the square described upon the two sides containing the given angle, considered as one line, exceeds the square described upon the third side, have a given ratio to the triangle.

This Proposition is demonstrated four different ways.

The First Demonstration.

Let ABC be a triangle, which has the angle ABC given; I say, the excess by which the square described upon AB and BC, considered as one line, exceeds the square of the third side AC, has a given ratio to the triangle ABC.

For produce AB, and cut a part of it, as BD, equal to BC^k; join DC, which produce at pleasure through the point A; draw AE parallel to BC^l; then, because the angle ABC is given, wherefore^m the angle CBD is given, consequentlyⁿ the sum of



^kProp. 3.
Euc. lib. 1.

^lProp. 31.
Euc. lib. 1.

^mProp. 13.
Euc. lib. 1.

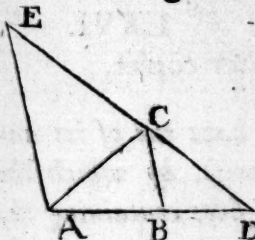
& Dat. 4.
ⁿProp. 32.
Euc. lib. 1.

the & Dat. 4.

the two angles BCD and BDC is given; but the triangle BCD is isoscelous, therefore each

• Prop. 5. of the angles BCD and BDC are given°,
 Euc. lib. 1. wherefore ^p the triangle
 & Dat. 4. CBD is given in species;
^p Dat. 40. and the ratio of BD to DC

^q Dat. 50.



is given, wherefore ^q the ratio of the square of BD to the square of DC is gi-

^r Prop. 2. ven. Again, because BC is parallel to AE^r,
 Euc. lib. 6. as AB is to BD, so is EC to CD; but as AB

^r Prop. 1. AB and BD, to the square of BD^s; and, for
 Euc. lib. 6. the same reason, EC is to CD, as the rectan-
 gle, contained between EC and CD, is to the
 square of CD; therefore, as the rectangle, con-
 tained between AB and BD, is to the square of
 BD, so is the rectangle, contained between EC
 and CD, to the square of CD; and by alterna-

^r Prop. 16. tion^t, as the rectangle, contained between AB
 Euc. lib. 5. and BD, is to the rectangle contained between
 EC and CD, so is the square of BD to the
 square of CD; but the ratio of the square of
 BD to the square of CD is given, therefore the
 ratio of the rectangle, contained between AB
 and BD, to that contained between EC and
 CD is given; but BD is equal to BC, therefore
 the ratio of the rectangle contained between
 AB and BC, containing the given angle to the
 rectangle contained between EC and CD, is

^t Dat. 65 given; but ^t the ratio of the rectangle, con-
 tained

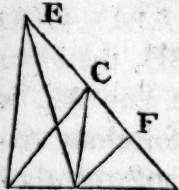
tained between AB and BC, to the triangle
 ABC, is given, therefore ^u the ratio of the ^u Dat. 8.
 rectangle, contained between EC and CD, to
 the triangle ABC, is given. Again, because
 BC is parallel to AE, the angle CBD is equal
 to the angle EAD ^w, and the angle ADE is ^w Prop. 29.
 common, wherefore the triangles EAD, CBD ^{Euc. lib. 1.}
 are similar ^x; but CB is equal to BD, therefore ^x Prop. 4.
 EA is equal to AD. Hence the line AC is ^{Euc. lib. 6.}
 drawn from the vertex of the isoscelous tri-
 angle, cutting the base at C, consequently ^y Lemma
 the rectangle contained between EC and CD,
 together with the square of AC, are equal to
 the square of AD; but AD is equal to AB,
 BC, taken together, wherefore the rectangle,
 contained between EC and CD, is the excess
 by which the square of AB, BC, considered as
 one line, exceeds the square of the third side
 AC; but the ratio of that rectangle to the
 triangle ABC is given, by the above demon-
 stration.

*Therefore, if any angle of a triangle be given,
 then will the excess, by which the square described
 upon the two sides containing the given angle,
 considered as one line, exceeds the square described
 upon the third side, have a given ratio to the tri-
 angle; which was to be demonstrated.*

The Second Demonstration.

Let the construction remain the same as before, from the point B draw BF perpendicular

^a Prop. 12. to CD², and join BE; then, because the angle ABC is given^a, the angle CBD is given^b, and consequently the angles BDC and BCD are each given; but the angle DFB is a right angle, wherefore the



^c Prop. 32. remaining angle FBD is given^c, and consequently^d the triangle BFD is given

^d Dat. 40. in species; wherefore^e the ratio of BF to FD is given. For the same reason, the ratio of

^f Dat. 8. BF to CF is given, therefore^f the ratio of DF to FC is given, and consequently^g the ratio of

^g Dat. 6. DC to DF is given; but the ratio of DF to BF is given, wherefore^h the ratio of BF to CD is

given; but as BF is to CD, so is the rectangle, contained between BF and EC, to the rectangle contained between DC and CEⁱ, therefore

ⁱ Prop. 1. the ratio of the rectangle, contained between BF and EC, to the rectangle contained between EC and CD, is given; but the rectangle contained between EC and CD, is the excess by

which the square of AB and BC, considered as one line, exceeds the square of the third side AC, by what has already been demonstrated,

and the rectangle contained between BF and EC is equal to double the triangle BCE^k, there-

^k Prop. 41. there-

^k Prop. 41. there-

therefore the ratio of the excess by which the square of AB and BC, considered as one line, exceeds the square of AC, to twice the triangle BCE, is given, and consequently the ratio of that excess, to the triangle BCE, itself is given; but the triangle BCE is equal to the triangle ABC¹, therefore the ratio of the excess by which the square of AB and BC, considered as one line, exceeds the square of the third side AC, to the triangle ABC, is given.

¹ Prop. 37.
Euc. lib. 1.

Therefore, &c. which was to be demonstrated.

The Third Demonstration.

The given angle ABC must either be a right angle, an acute angle, or an obtuse angle.

First, let it be a right angle: then will the square of AB and BC, considered as one line, be greater than the two squares of AB and BC, by twice the rectangle contained between AB and BC^m; but the two squares AB and BC are equal to the square



^m Prop. 4.
Euc. lib. 2.

of ACⁿ, therefore twice the rectangle, contained between AB and BC, is equal to the excess by which the square of AB and BC, considered as one line, exceeds the square of the third side AC; but the rectangle, contained between AB and BC, has a given ratio to the triangle ABC^o, wherefore twice the rectangle, contained between AB and BC, has a given ratio to the triangle ABC.

ⁿ Prop. 47.
Euc. lib. 1.

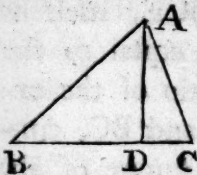
Again,

• Dat. 65.

Again, let the given angle ABC be an acute angle: from the point A draw the line AD ,

^p Prop. 12. perpendicular to BC ^p; then, because the tri-
 Euc. lib. 1. angle ABC is an acute angled triangle, and

from the angular point A , AD is drawn perpendicular to BC , therefore^q the two squares, described upon AB and BC , exceed the square of AC , by



^q Prop. 13.
 Euc. lib. 2.

twice the rectangle contained between BC and BD ; but the square of AB and BC , considered as one line, exceeds the two squares of AB and BC , by twice the rectangle contained between AB and BC ^r, therefore the square of AB and BC , considered as one line, exceeds the square of AC , by twice the rectangle contained between BC and BD ; together with twice the rectangle contained between AB and BC , that is, twice the rectangle contained between AB and BD , considered as one line, and the line BC . Again, because the angle ABD is given, and ADB is a right angle, the remaining third

^r Prop. 32. angle BAD is given^r, and consequently^t the
 Euc. lib. 1. triangle ABD is given in species; wherefore^u

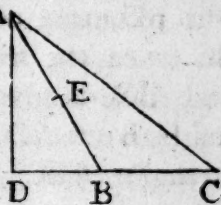
& Dat. 4. the ratio of AB to AD is given, and the ratio
^t Dat. 40. of BD to AD is given, therefore^w the ratio of
^u Def. 3. AB , BD , considered as one line, to AD is gi-
 of this. ven; but as AB , BD , considered as one line, is

^w Dat. 22. to AD , so is the rectangle contained between AB , BD , as one line, and the line BC , to the

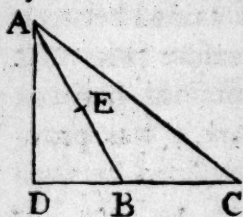
^x Prop. 1. rectangle contained between AD and BC ^x,
 Euc. lib. 6. there-

therefore the ratio of the rectangle contained between AB, BD, considered as one line, and the line BC, to the rectangle contained between AD and BC, is given, and therefore twice that rectangle to the rectangle contained between AD and BC is given: but before it was proved, that twice the rectangle contained between AB, BD, considered as one line, and the line BC, is the excess by which the square of AB, BC, considered as one line, exceeds the square of AC; and the rectangle contained between AD and BC is equal to double the triangle ABC^y, therefore the ratio of that excess to double the triangle, and consequently to the triangle ABC, itself is given. ^{Prop. 41. Euc. lib. 1.}

Lastly, when the given angle ABC is an obtuse angle: from the point A draw AD perpendicular to CB^z, which cuts it produced at ^{Prop. 12. Euc. lib. 1.} D, and make BE equal to A
BD; then, because the two squares of AB and BC are smaller than the square of AC, by twice the rectangle contained between DB and BC^a, that is, by twice the rectangle contained ^{Prop. 12. Euc. lib. 2.} between EB and BC; but the square of AB, BC, considered as one line, is greater than the two squares described upon AB and BC, by twice the rectangle contained between AB and BC^b, therefore the square of AB, BC, con- ^{Prop. 4. Euc. lib. 2.} sidered



dered as one line, exceeds the square of AC, by twice the rectangle contained between AE



^c Prop. 13.
Euc. lib. 1.

and BC. Again, because the angle ABC is given, wherefore ^c the angle ABD is given, and the angle ADB is a right angle, therefore the remaining angle DAB ^d

^d Prop. 32.
Euc. lib. 1.

is given, and consequently ^e the triangle ADB is given in species; wherefore ^f the ratio of AD to AB, and also the ratio of AD to DB or BE, which is equal to it, is given, wherefore ^g the ratio of AD to AE is given; but as AD is to AE, so is the rectangle, contained between AD and BC, to the rectangle contained between AE and BC ^h; therefore the ratio of the rectangle, contained between AD and BC, to the rectangle contained between AE and BC is given, and consequently the ratio of the rectangle contained between AD and BC, to twice the rectangle contained between AE and BC, is given; but the rectangle, contained between AD and BC, is equal to double the triangle ABC ⁱ, and twice the rectangle, contained between AE and BC, has been proved to be the excess by which the square AB, BC, considered as one line, exceeds the square of the third side AC; wherefore the ratio of that excess, to double the triangle ABC is given, and consequently the ratio of that excess to the triangle ABC itself is given.

^e Dat. 40.
^f Def. 3.
of this.

^g Dat. 5.
and 8.

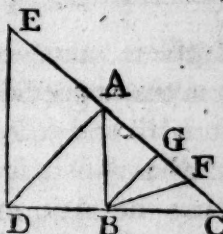
^h Prop. 1.
Euc. lib. 6.

ⁱ Prop. 41.
Euc. lib. 1.

*Therefore, if any right lined triangle, &c,
which was to be demonstrated.*

The Fourth Demonstration.

Produce CB, and make BD equal to BA;
from the points B and D, draw the Lines DE,
BF making the angles DEA,
BFA, each equal to the angle
BAD, ^k and draw GB per-
pendicular to AC, cutting it
in G^l; then, because BA is
equal to BD, ^m each of the
angles BAD and BDA are equal to half the
angle ABC; but the angle ABC is givenⁿ,
wherefore each of the angles BAD, ADB are
given, and consequently ^o the angle ABD is
given; wherefore the triangle ABD is given in
species^p, and the ratio of DA to AB is given.
Again, because the angle DEC is equal to the
angle ADC^q, and the angle DCE is common,
therefore^r the remaining angle EDC is equal
to the remaining angle DAC: and^s as EC is
to CD, so is CD to CA; wherefore^t the rectan-
gle contained between EC and CA is equal to
the square of DC, that is the square of AB, BC
considered as one line. But the rectangle con-
tained between EC and CA exceeds the square
of AC, which is the third side of the first trian-
gle by the rectangle contained between EA and
AC^v; wherefore the rectangle contained between



^k Above
Problem,

^l Prop. 12.
Euc. lib. 1.

^m Props. 5,
and 32.

ⁿ Hypoth.
Euc. lib. 1.

^o Prop. 32.
Euc. lib. 1,

& Dat. 4.
^p Dat. 40.

^q Construc-
tion.

^r Prop. 32.
Euc. lib. 1.

^s Prop. 4.
Euc. lib. 6,

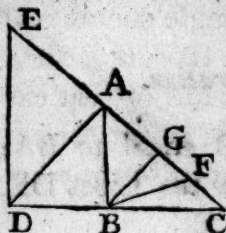
^t Prop. 17.
Euc. lib. 6,

^v Prop. 3.
Euc. lib. 3.

I

EA

EA and AC is the excess by which the square of AB, BC, considered as one Line, exceeds the square of the third side AC.



Again, because the angle EDC is equal to the angle DAC, and the angle ADB equal to the angle DAB, therefore the remaining angle

BAC is equal to the remaining angle EDA;

• Construc-
tion.

• Prop. 32.
Euc. lib. 1.

• Prop. 4.
Euc. lib. 6.

but the angle DEA is equal to BFA^w, wherefore^x the angle ABF is equal to the angle EAD; and^y as AD is to AB, so is EA to FB; but the ratio of AD to AB is given, wherefore the ratio of EA to BF is given. Again, because the angle BFG is given, and the angle BGF is

• Prop. 32.
Euc. lib. 1.

& Dat. 4.
• Dat. 40.

• Def. 3.
Dat.

a right angle, wherefore^z the remaining angle GBF is given, and consequently^a the triangle BGF is given in species, wherefore^b the ratio of BF to BG is given; but before it was proved that the ratio of EA to BF was given, therefore

• Dat. 8.

^c the ratio of EA to BG is given. But as EA is to BG, so is the rectangle contained between EA and AC to the rectangle contained between

• Prop. 1.
Euc. lib. 6.

BG and AC^d, wherefore the ratio of the rectangle contained between EA and AC to the rectangle contained between BG and AC is given, and consequently the ratio of twice the rectangle contained between EA and AC to the rectangle contained between BG and AC is given; but before we proved twice the rectangle contained between EA and AC equal to the

Excess by which the square of AB and BC, considered as one line, exceeds the square of the third side AC; and the rectangle contained between BG and AC is ^e equal to double the triangle ABC; wherefore the ratio of that excess to double the triangle ABC is given, and consequently the ratio of that excess to the triangle ABC itself is given.

^e Prop. 41,
Euc. lib. 1.

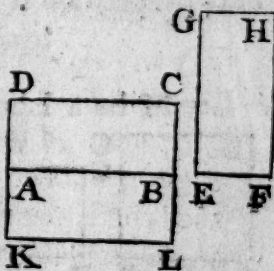
Therefore, if any one angle of a triangle be given, &c. which was to be demonstrated.

PROPOSITION LXVII.

or 68th in other copies.

If two equiangular parallelograms have a given ratio to one another, and if one side in the one have a given ratio to one side in the other, the remaining side in the one will have a given ratio to the remaining side in the other.

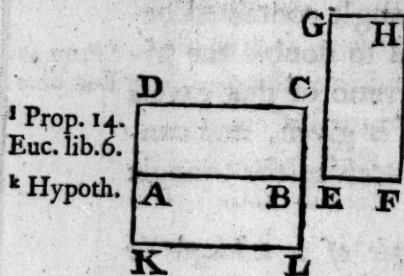
Let AC, EH be two equiangular parallelograms which have a given ratio to one another, and let the side AB have a given ratio to the side EF; I say the side BC will also have a given ratio to the side EG. For produce the side DA, and construct the parallelogram AL equal to



EH^f. Then because the parallelogram AC has a given ratio to EH^g, and AL is equal to EH, therefore AC has a given ratio to AL; but

^f Prop. 43.
Euc. lib 1.
^g Hypoth,

^h Prop. 1. as AC is to AL, so is CB to BL^h, wherefore
 Euc. lib. 6. the ratio of CB to BL is given. Again, be-



¹ Prop. 14.
 Euc. lib. 6.

^k Hypoth.

cause AL is equal to EH, and they are equiangular, wherefore¹ as AB is to EF, so is EG to BL; but the ratio of AB to EF is given^k, wherefore the ratio of EG to BL is given; but before

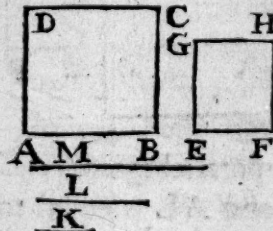
it was proved that the ratio of CB to BL was given, and now the ratio of GE to BL is given, wherefore¹ the ratio of BC to EG is given.

¹ Dat. 8.

Therefore, if two equiangular parallelograms have a given ratio to one another, and if one side in the one, has a given ratio to one side in the other, the remaining side in the one will have a given ratio to the remaining side in the other; which was to be demonstrated.

This proposition is also demonstated in the following manner.

Let M be a Line of any given magnitude, and since the ratio of the parallelogram AC to the parallelogram EH is given, let M be to K as AC is to EH, wherefore the ratio of M to K is given; but



^m Def. 2. M is given, therefore^m K is given. Again, since

since the ratio of AB to EF is given, let the ratio of M to L be equal to it, then is the ratio of M to L given; but M is given, wherefore ^a Dat. 2. L is given: now since L is given, and K also given, wherefore ^o the ratio of L to K is given. ^o Dat. 1. Again, because the parallelogram AC is equiangular to the parallelogram EH, therefore ^p Prop. 23. the parallelogram AC is to the parallelogram EH in a ratio compounded of AB to EF, and of BC to EG; but ^q M is to K, as AC is to EF, ^q Construction. therefore M is to K in a ratio compounded of AB to EF, and of BC to EG; but M is to K in a ratio compounded of M to L, and of L to K, wherefore the ratio compounded of AB to EF, and of BC to EG, is equal to that which is compounded of M to L, and of L to K; but the ratio of AB to EF is equal to the ratio of M to L, wherefore the ratio of BC to EG is equal to the ratio of L to K; but the ratio of L to K is given, wherefore the ratio of BC to EG is given.

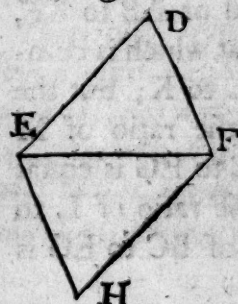
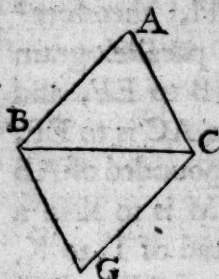
Therefore, if two equiangular parallelograms, &c. which was to be demonstrated.

COROLLARY.

From this proposition it follows, that if two triangles have a given ratio to one another, and if they have one angle in the one equal to one angle in the other, and if one of the sides in each triangle about the equal angles have a given ratio to

one another, the other side about the equal angle in the one triangle will likewise have a given ratio to the other side about the equal angle in the other triangle.*

Let ABC, DEF be two triangles, having a given ratio to one another, and let the angle



† Prop. 34.
Euc. lib. I.
† Prop. 15.
Euc. lib. 5.

BAC be equal to the angle EDF, and let the ratio of AB to DE be given; I say the ratio of the other side AC, in the triangle ABC, to the other side DF, in the triangle EDF is also given. For complete the parallelograms AG, DH; then, because the triangles ABC, EDF are each half their respective parallelograms AG, DH†, therefore † as the triangle ABC is to the triangle EDF, so is the parallelogram AG to the parallelo-

gram DH; but the ratio of the triangle ABC to the triangle EDF is given†, wherefore the ratio of the parallelogram AG to the parallelogram DH is given, and the angle BAC is equal to EDF, and the ratio of AB to DE is given, therefore by the above proposition the ratio of AC to DF is given.

* This corollary is not in the other editions.

PRO-

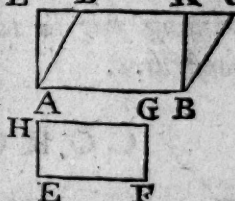
PROPOSITION LXVIII.

or 69th in other copies.

If two parallelograms have their angles given, and have a given ratio to one another, and if one side of the one have a given ratio to one side of the other, the other side in the one, will likewise have a given ratio to the remaining side in the other.

Let AC and EG be two parallelograms which have the angles DAB, HEF given, and let AC have a given ratio to EG, L D K C

and let the side AB have a given ratio to EF; I say the remaining side BC will likewise have a given ratio to the remaining FG. First, if the



angle DAB be equal to the angle HEF, the proposition in that Case is manifest from the 67th Data; but if the angle DAB is not equal to the angle HEF, with the line BA, and at the point

A, construct the angle BAL, equal to FEH^v, and compleat the parallelogram AK^w; then, because the ratio of AC to EG is given^x, and AC is equal to the parallelogram AK^y; wherefore the ratio of AK to EG is given; but AK is equiangular to EG^z, and AB has a given ratio to EF^a, wherefore^b the remaining side BK has a given ratio to the side FG. Again, because the angles ABC, ABK are each given^c, therefore^d the angle KBC is given, and the angle KCB is

^v Prop. 23.
^w Prop. 31.
^x Hypoth.
^y Prop. 35.
^z Constr.
^a Hypoth.
^b Dat. 67.
^c Hypoth.
^d Dat. 1.

* Prop. 32. given, wherefore^e the remaining angle BKC is
 Euc. lib. 1. L D K C given, and consequently^f
 & Dat. 4. the triangle BKC is given in
 † Dat. 40. species, wherefore^g the ratio
 ‡ Def. 3. of BC to BK is given; but
 Dat. it has been proved that the
 ratio of BK to EG is given;



‡ Dat. 8. consequently^h the ratio of BC to EG is given.

Therefore, if two parallelograms have a given ratio to one another, and if one side of the one have a given ratio to one side of the other, the other side in the one will likewise have a given ratio to the remaining side in the other; which was to be demonstrated.

C O R O L L A R Y.

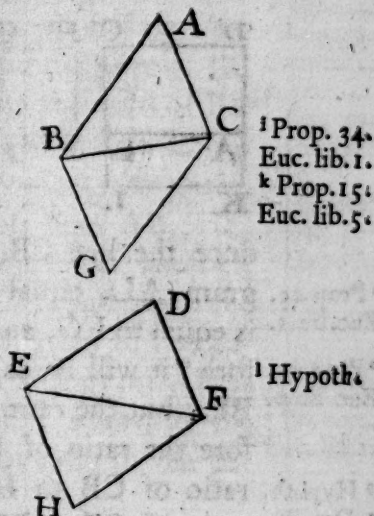
From this proposition it follows, if two triangles have a given ratio to one another, and if one angle in each of them be given, and if one side about the given angle in the one has a given ratio to one side about the given angle in the other, then will the remaining side about the given angle in one have a given ratio to the remaining side about the given angle in the other triangle.*

Let ABC, DEF be two triangles which have a given ratio to one another, and let the angles BAC, EDF be each given in magnitude, and let the side AB have a given ratio to the side DE, then will the other side AC have likewise

* *This corollary is not in the other editions.*

a given

a given ratio to the other side DF. For complete the two parallelograms AC, DH; then, because the triangles ABC, DEF are each the half of the parallelograms AG, DH, respectivelyⁱ, therefore^k as the triangle ABC is to the triangle DEF, so is the parallelogram AG to the parallelogram DH; but the ratio of the triangle ABC to the triangle DEF is given^l, therefore the ratio of the parallelogram AG to the parallelogram DH is given, and the angles BAC, EDF are each given in magnitude, and the ratio of AB to DE is given^m; therefore by the above propositionⁿ Hypoth. the ratio of the remaining side AC to the remaining side DF is also given.



ⁱ Prop. 34.
Euc. lib. 1.
^k Prop. 15.
Euc. lib. 5.

^l Hypoth.

ⁿ Hypoth.

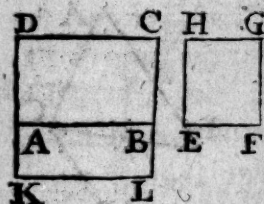
PROPOSITION LXIX.

or 70th in other copies.

If the sides of two equiangular parallelograms, or two parallelograms whose angles are given, though not equal, have given ratios to one another, each to each, these parallelograms will also have a given ratio to one another.

Let AC, EG be two parallelograms which are either equiangular, or have their angles given, though

though not equal, and let the sides AB , EF have a given ratio to one another; also let the



sides BC , EH have a given ratio to one another; I say the parallelogram AC has a given ratio to the parallelogram EG . First let them be equiangular: pro-

duce the line CB , and compleat the parallelo-

^a Prop. 45. gram AL , equal to EG^a ; then, because AL is equal to EG , and they are equiangular, there-

^b Prop. 14. fore ^c it will be as AB is to EF , so is EH to BL ; but the ratio of AB to EF is given, where-

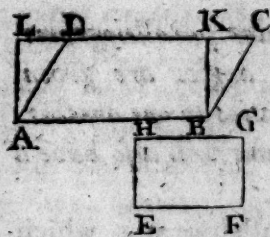
^d Hypoth. fore the ratio of EH to BL is given; but the ratio of CB to HE is given^e, wherefore^f the

^g Dat. 8. ratio of CB to BL is given. But as CB is to BL , so is the parallelogram AC to the paral-

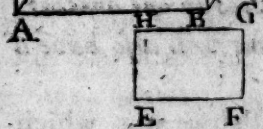
^h Prop. 1. lelogram AL^h , wherefore the ratio of the paral-

ⁱ Euc. lib. 6. lelogram AC to the parallelogram AL is given; but AL is equal to EG^i , consequently the ratio of the parallelogram AC to the parallelogram EG is given.

But if the angle ABC is not equal to the angle EFG , with the line



^j Prop. 23. Euc. lib. 1.



AB and at the point B construct the angle ABK equal to EFG^j , and compleat the parallelogram AK ; then, because the angles ABC , ABK are each

^k Hypoth. given^k, the angle KBC is given^k, and the angle

gle

gle BCK is given^a, wherefore the remaining^a Prop. 29.
 angle BKC^r is given, consequently^a the trian- Euc. lib. 1.
 gle BKC is given in species, wherefore^a the & Dat. 4.
 ratio of BC to BK is given; but the ratio of BC Prop. 32.
 to FG is given^b, wherefore^c the the ratio of BK Euc. lib. 1.
 to FG is given; but the ratio of AB to EF is & Dat. 4.
 given^d, hence there are two equiangular paral- z Dat. 40.
 lelograms AK and EG, whose sides AB, EF, a Def. 3.
 BK, and FG, have given ratios to one another, Dat.
 each to each; by the above demonstration the b Hypoth.
 parallelogram AK has a given ratio to the pa- c Dat. 8.
 rallelogram EG; but the parallelogram AK is d Hypoth.
 equal to the parallelogram AC^e, wherefore the e Prop. 35.
 parallelogram AC has a given ratio to the pa- Euc. lib. 1.
 rallelogram EG.

Therefore, if the sides of two equiangular parallelograms, or two parallelograms whose angles are given, though not equal, have given ratios to one another, each to each, these parallelograms will also have a given ratio to one another; which was to be demonstrated.

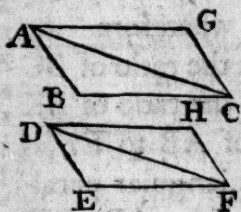
COROLLARY TO PROPOSITION.

This is PROPOSITION 72 in the other Copies.

If two triangles have one angle of the one equal to one angle of the other, or one angle in each given, though not equal, and if the side about the equal or given angles have given ratios to one another, each to each, the triangles will also have a given ratio to one another.

Let

Let there be two triangles ABC and DEF , which have the angles ABC , DEF equal to one another, or not equal, but given, and let AB have a given ratio to DE , and BC a given ratio to EF ; I say the triangle BAC will have a given ratio to the triangle



DEF . For compleat the parallelograms BG and EH : then because the parallelograms BG and EH have the angles ABC , DEF equal to one another, or not equal, but given, and the sides about the equal angles have a given ratio to one another, each to each, by the above proposition the parallelogram AG has a given ratio to the parallelogram EH ; but the triangles ABC , DEF are each half the parallelograms BG , EH ⁱ; therefore as the triangle ABC is to the triangle DEF , so is the parallelogram BG to the parallelogram EH ; but the ratio of the parallelogram BG to the parallelogram EF is given, therefore the triangle ABC has a given ratio to the triangle DEF .

ⁱProp. 34.
Euc. lib. 1.
and Prop.
15. Euc.
lib. 5.

PROPOSITION LXX.

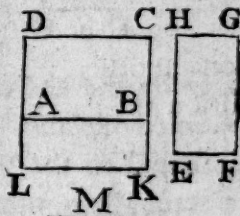
This Proposition is the 73d in other copies.

If two parallelograms be equiangular, or not equiangular, but have their angles given, and if one side of one of them have a given ratio to that right line which is a fourth proportional to the other

other side of the same parallelogram, and the two sides of the other parallelogram, these parallelograms will have a given ratio to one another.

Let AC, EG be two parallelograms, either equiangular, or which have their angles given, and let M be a fourth proportional to AB, EF, and EH, and let BC have a given ratio to M; I say the parallelogram AC will have a given ratio to the parallelogram EG.

For first let the parallelograms AC, EG be equiangular; produce BC, and make BK equal to M, and compleat the parallelogram AK; then, because AB is to EF, as EH is to M^z, but BK is equal to M, therefore as AB is to EF, so is EH to BK; and



the angle ABK is equal to GHE, wherefore the parallelogram AK is equal to the parallelogram EG^h. But as CB is to BK, so is the parallelogram AC to the parallelogram AKⁱ; but the ratio of BC to BK is given^k, wherefore the ratio of the parallelogram AC to the parallelogram AK is given; but AK is equal to EG, wherefore the ratio of the parallelogram AC to the parallelogram EG is given.

Again, if the parallelograms AC, EG are not equiangular, but the angles ABC, HEF are given; then, with the line AB, and at the point B, construct the angle ABK, equal to HEF^l, and compleat the parallelogram AK^m;

then

z Construction.

^h Prop. 14.

Euc. lib. 6.

ⁱ Prop. 1.

Euc. lib. 6.

^k Hypoth.

^l Prop. 28.

Euc. lib. 1.

^m Prop. 31.

Euc. lib. 1.

then, because the angles ABC, ABK are each
 a Dat. 4. given by the Hypothesis, therefore^a the re-
 ° Prop. 29. maining angle KBC is given; but ° the angle
 Euc. lib. 1. BCK is given, wherefore^b the remaining third
 & Dat. 4. angle BKC is given, consequently^c the triangle
 Prop. 32. BKC is given in species, where-
 Euc. lib. 1. fore^d the ratio of BC to BK is
 & Dat. 4. given; but^e the ratio of BC to
 ° Dat. 40. HE is given, wherefore^f the
 ° Def. 3. ratio of BK to EH is given.
 Dat. But the ratio of AB to EF is
 ° Hypoth. given^g, consequently the equi-
 ° Dat. 8. angular parallelograms AK,
 ° Hypoth. EG by the above demonstration have a given
 ratio to one another; but the parallelogram AC
 is equal to the parallelogram AK^h, wherefore
 the parallelogram AC has a given ratio to the
 parallelogram EG.

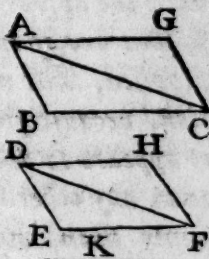
Therefore, if two parallelograms be equiangular, or not equiangular, but have their angles given, and if one side of one of them have a given ratio to that right line which is a fourth proportional to the other side of the same parallelogram, and the two sides of the other parallelogram, these parallelograms will have a given ratio to one another, which was to be demonstrated.

COROLLARY TO PROPOSITION 70.

If two triangles have one angle in the one equal to one angle in the other, or not equal, but given in

in magnitude, and if one of the sides, about the equal or given angles in the one, have a given ratio to that line which is a fourth proportional to the other side, about the equal or given angle, in that first mentioned triangle, and the two sides in the other triangle, which are about the equal or given angle in it, the triangles will have a given ratio to one another.

Let the two triangles be ABC, DEF, which have the angles ABC, DEF either equal, or not equal, but given, and let BC be to DE, as EF is to K, and let AB have a given ratio to K, the triangle ABC will have a given ratio to the triangle DEF.



For complete the parallelograms BG and EH^{*}; then, because the angles ABC, DEF

^{*} Prop. 31.
Euc. lib. 1.

are either equal, or not equal, but given, and AB has a given ratio to K, which is a fourth proportional to BC, DE, and EF, therefore, by the above proposition the parallelogram BG has a given ratio to the parallelogram EH, and consequently [†] the triangle ABC has a given ratio to the triangle DEF^{*}.

[†] Prop. 15.
Euc. lib. 5.

There is no series in the whole Data more corrupted than this. Pappus says it consists of seven propositions, viz. from the sixty seventh to the

^{*} *The above corollary is not in the other editions of the Data.*

seventy

seventy third inclusive, which, he informs us, treat of two parallelograms, which are supposed to have equal or given angles, that the parallelograms themselves have given ratios to one another, &c. and some, he tells us, add to these as corollaries, the same properties of triangles; and though nothing can be more explicit than that the whole series demonstrate certain properties of parallelograms; yet in all the editions there are no less than eleven, and in the whole there are only six that have the appearance of those mentioned by Pappus, so that one of the seven is wanting; this we have supplied, and made it the seventy second proposition, because from the property demonstrated by it, it appears to be more properly ranged in that place than any other. One of the eleven, viz. the seventy sixth, evidently belongs to the next series; and the seventy seventh is a part of the fifty fourth, which we have there inserted, as has already been taken notice of in our note to that proposition. Three of them are concerning triangles, and because they demonstrate the same properties of triangles which three in this series do of parallelograms, they are three of those which Pappus mentions were added by some to the propositions in this series as corollaries. And hence, by adding the above mentioned proposition, and by taking away these three which treat of triangles, and the seventy sixth and seventy seventh, which do not belong to it, and by adding the same properties of triangles as corollaries to their respective propositions, the series, by that

I

means

means, exactly corresponds to that account of it given by Pappus: And since in the other copies there are no more than three of these properties of triangles to be found, we have supplied the other four. Those which are not in the other copies are taken notice of in a note to their respective propositions.

PROPOSITION LXXI.

which in other copies is 74.

If two parallelograms be equiangular, or not, but have their angles given, and if these parallelograms have a given ratio to one another, then will one side of one of them have a given ratio to that right line, which is a fourth proportional to the other side of the same parallelogram, and the two sides of the other parallelogram.

This proposition is the converse of the seventieth, as the sixty ninth is the converse of the sixty seventh and sixty eighth; but as the first part of the proposition is the same, and demonstrated in the same manner, step by step, with the fifty sixth proposition, we cannot help concluding, that either the fifty sixth is spurious, or this is corrupted: we shall therefore pass over the first part, and proceed to the second, to wit,

When the angles ABC, EFG are not equal, but given; then, with the line AB, and at the point B, construct the angle ABK, equal to EFG², and compleat the parallelogram ^{= Prop. 23.}

K

AK²; Euc. lib. I.

^a Prop. 31. AK^a ; then because the angles ABC , ABK are
^b Euc. lib. 1. each given ^b, wherefore ^c the angle KBC is
^c Hypoth. given; and since the angle ABC is given, the
^d Dat. 4. angle BCK is given ^d, wherefore ^e the remaining

^e Prop. 29. Euc. lib. 1.

& Dat. 4.

^f Prop. 32.

Euc. lib. 1.

& Dat. 4.

^f Dat. 40.

^g Def. 3.

Dat.

^h Prop. 35.

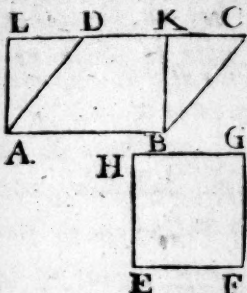
Euc. lib. 1.

ⁱ Constr.

tion.

^k Dat. 56.

^l Dat. 8.



angle BKC is given, and consequently the triangle BKC^f is given in species, and therefore ^g the ratio of BK to BC is given. Again, because the parallelogram AK is equal to AC^h , and AC has a given ratio to EG , therefore AK

has a given ratio to EG ; but ⁱ AK is equiangular to EG , wherefore ^k KB has a given ratio to that line which is a fourth proportional to AB , EF , and FG ; but we have proved that KB has a given ratio to BC , wherefore ^l BC will have a given ratio to that right line, which is a fourth proportional to AB , EF , and FG .

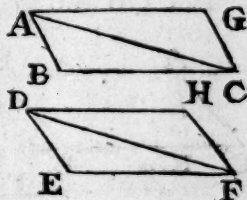
Therefore, if two parallelograms be equiangular, or not, but have their angles given, and if these parallelograms have a given ratio to one another, then will one side of one of them have a given ratio to that right line, which is a fourth proportional to the other side of the same parallelogram, and the two sides of the other parallelogram; which was to be demonstrated.

COROLLARY.

This Corollary is Proposition 75 in the other copies.

If two triangles have a given ratio to one another, and if they have one angle of the one equal to one angle of the other, or not equal, but given, then will one side of one of the triangles, about the equal or given angle, have a given ratio to that right line, which is a fourth proportional to the other side in the same triangle, about the equal or given angle, and the two sides of the other triangle, which are about the equal or given angle.

Let ABC, DEF be two triangles which have a given ratio to one another, and let the angle ABC be either equal to the angle DEF, or not equal, but given; then will one side of one of them, as AB,



have a given ratio to that line, which is a fourth proportional to the other line BC, and the two lines in the other, about the equal or given angle, to wit DE and EF. For compleat the parallelograms BG and EH; then, because the triangle ABC has a given ratio to the triangle DEF, therefore^m the parallelogram BG has a^m given ratio to the parallelogram EH, and the angles ABC, DEF are either equal, or not equal, but given; wherefore by the above pro-

^mProp. 15.
Euc. lib. 5.

position AB will have a given ratio to that line, which is a fourth proportional to BC, DE, and EF.

PROPOSITION LXXII.

according to Pappus. This is not in any of the copies, but is here supplied.

If there be two parallelograms, and if from any point in one side of each parallelogram lines be drawn, making either equal angles with their opposite sides, or angles not equal, but given, and if these lines, and also the sides from or to which they are drawn, have given ratios to one another, the parallelograms will have a given ratio to one another.

Let CB and GF be two parallelograms, and

Fig. 1.

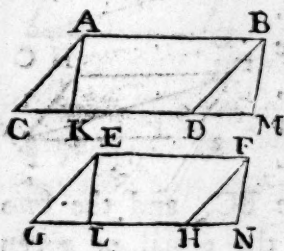
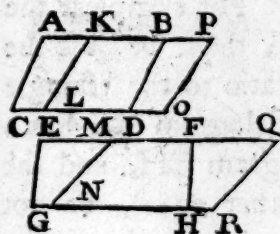


Fig. 2.



from any point in each of the lines AB, EF, as A and E, fig. 1, or K and M, fig. 2, let the lines AK, EL, fig. 1, or KL, MN, fig. 2, making equal angles, or not equal, but given angles, with the lines AB, CD, EF, and GH, and let AK have a given ratio to EL, fig. 1, and AB to EF, and in fig. 2 let KL have a given ratio to MN, and AB a given ratio to EF;

EF; If say the parallelogram CB has a given ratio to the parallelogram GF. For the assumed points will either be in the angular points of the parallelograms, or not. First let them be in the angular points, as in fig. 1; upon the bases AB, EF compleat the parallelograms MA, EN^p. But, if these points are not in the angular points of the parallelograms, but at K and M, fig. 2, produce the lines AB, EF, and make KP, MQ equal to AB and EF, each to each, and compleat the parallelograms KO and MR; then, because the two parallelograms AM, EN, fig. 1, and KO, MR, fig. 2, are either equiangular, or not, but have their angles given, and the sides about the equal or given angles have given ratios to one another, each to each, wherefore the parallelograms AM, EF, fig. 1, and KO, MR, fig. 2, have also given ratios to one another^a; but the parallelograms MA, EN are equal to the given parallelograms CB, GF^p, and the parallelograms KO, MR, fig. 2. are equal to the given parallelograms CB, GF, each to each^a, wherefore the parallelogram CB has a given ratio to the parallelogram GF.

^a Prop. 31.
Euc. lib. 1.

• Dat. 69.

^p Prop. 35.
Euc. lib. 1.

^a Prop. 36.
Euc. lib. 1.

Therefore, if there be two parallelograms, and if from any point in one side of each parallelogram lines be drawn, making either equal angles with their opposite sides, or angles not equal, but given, and if these lines, and also the sides from or to which they are drawn, have given ratios to one

EUCLID'S DATA.

another, the parallelograms will have a given ratio to one another; which was to be demonstrated.

COROLLARY I.

The same things remaining, it follows, that if the parallelograms have a given ratio to one another, and if these lines, drawn from a point in one side of each parallelogram, make equal angles with the other side, or angles not equal, but given; then, the sides, from which these lines are drawn, will also have a given ratio to one another; and if these sides have a given ratio to one another, and the parallelograms a given ratio to each other, these lines, drawn from the assigned points in their sides, will also have a given ratio to one another *.

COROLLARY II.

This is PROPOSITION 72 in the other Copies.

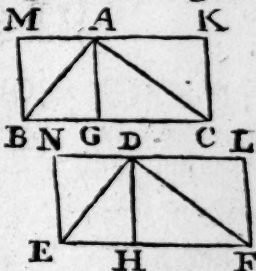
If the bases of two triangles have a given ratio, and if lines drawn from their vertexes, making either equal angles with their bases, or given angles, though not equal, have also a given ratio to one another, these triangles will have a given ratio to one another.

Let there be two triangles, as ABC and DEF, whose bases BC and EF have a given ratio to

* *This corollary is not in any of the other copies.*

one

one another, and from the vertexes A and D let the lines AG and DH be drawn, cutting the bases, either at equal angles, or at given angles, though not equal; and let these lines AG, DH have a given ratio to one another; then will the triangle ABC have a given ratio to the triangle DEF.



For, through the points A and D draw the lines AK, DL, parallel to the bases BC and EF, and through the points B and E draw the lines BM and EN, parallel to AG and DH, and compleat the parallelograms BK, EL; then, because from the points A and D, taken in the sides of the parallelograms BK, EL, the lines AG, DH are drawn, making equal or given angles with the other sides BC, EF, and these lines AG, DH, BC, and EF, have given ratios to one another, each to each, therefore, by the above proposition, the parallelogram BK has a given ratio to the parallelogram EL, and consequently the triangle ABC has a given ratio to the triangle DEF.

^r Prop. 31.
Euc. lib. 1.

^r Prop. 41.
Euc. lib. 1.
& Pr. 15.
Euc. lib. 5.

PROPOSITION LXXIII.

which is the 73d by Pappus's account, and the 78th in other copies.

*If a figure given in species * has a given ratio to a rectangle, and one side of the figure has also a given ratio to one side of the rectangle, the rectangle is given in species.*

Let ABCD be any right lined figure given in species, and EG a rectangle, and let ABCD have a given ratio to EG, and also let the side AB have a given ratio to the side EF; I say the rectangle EG is given in species. For upon AB describe the square AL^t, produce HE, and compleat the rectangle EN, equal to the square AL^v; then, because AL is a square, it is given in species^w; wherefore

^t Prop. 46.
 Euc. lib. 1.
^v Prop. 45.
 Euc. lib. 1.
^w Def. 3.
 Dat.

* This proposition in other copies is enuntiated thus: *If a given figure has a given ratio to a rectangle, &c. which is a corruption; because here the magnitude is supposed to be given, whereas by the first step of the demonstration the species of the figure is the property which is supposed in the Hypothesis, and not the magnitude; on the contrary, if the magnitude was supposed to be given, the first step, and consequently the whole demonstration, would be false; therefore to free the proposition of ambiguity, and the demonstration of fallacy, we have altered the enuntiation.*

ABCD and AL are both given in species, and described upon the same line AB, therefore the figure ABCD has a given ratio to AL^a; but ABCD has a given ratio to EG^y, wherefore^z AL has a given ratio to EG; but AL is equal to EN, therefore EG has a given ratio to EN; but as EG is to EN, so is HE to EM^a, wherefore HE has a given ratio to EM.



^a Dat. 49.
^y Hypoth.
^z Dat. 8.

^a Prop. 1.
Euc. lib. 6.

Again, because AL is equal to EN^b, and equiangular to it^c, as AB is to EF, so is EM to BL or AB; but AB has a given ratio to EF^d, wherefore^e EM has a given ratio to AB; but EM has a given ratio to EH, wherefore^f HE has a given ratio to AB; but AB has a given ratio to EF, wherefore^g HE has a given ratio to EF; hence the parallelogram EG has all its angles given, being right angles, and its sides have a given ratio to one another, consequently the parallelogram EG^h is given in species.

^b Construction.

^c Prop. 14.
Euc. lib. 6.

^d Hypoth.
^e Dat. 8.

^f Dat. 8.

^g Dat. 8.

^h Def. 3.
Dat.

Therefore, if a figure given in species, has a given ratio to a rectangle, and one side of the figure has also a given ratio to one side of the rectangle, the rectangle is given in species; which was to be demonstrated.

COROL.

COROLLARY.

From hence it follows, that if a figure given in species has a given ratio to a right angled triangle, and one side in the figure hath also a given ratio to one of the sides about the right angle in the triangle, the triangle is given in species.

Let the figure ABCD, given in species, have a given ratio to the right angled triangle EFG,



right angled at F, and let the side AB have a given ratio to the side FG, the triangle EFG is given in species. For, complete the rectangle FH; then, because the ratio of the figure ABCD to the triangle EFG

is given; but the triangle EFG is half the parallelogram FH¹, wherefore the ratio of the figure ABCD to the rectangle FH is given, and the ratio of AB to FG is given; therefore by the above proposition the rectangle FH is given in species, and consequently the ratio of EF to FG is given; but the angle EFG is given, being a right angle, therefore the triangle EFG is given in species^k *.

* *This corollary is not in any of the other copies.*

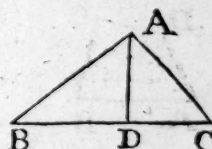
PROPOSITION LXXIV.

This is PROPOSITION 76 in other copies.

If a triangle be given in species, and from one of its angles a line be drawn perpendicular to its opposite side, that perpendicular has a given ratio to that side which it cuts.

Let ABC be a triangle given in species, and from the angular point A let AD be drawn perpendicular to BC; I say the line AD has a given ratio to BC.

Because the triangle ABC is given in species¹, therefore the angle ABD is given^m; and the angle ADB is a right angle, whereforeⁿ the remaining angle BAD is given, and consequently || the triangle ADB is given in species, wherefore^o the ratio of AD to DB is given; for the same reason the ratio of AD to DC is given, wherefore^p the ratio of BD to DC is given, and consequently^q the ratio of BC to DC is given; but the ratio of AD to DC is given, wherefore^r the ratio of AD to BC is given.



¹ Hypoth.
^m Def. 3.
ⁿ Dat.
^o Prop. 32.
^p Euc. lib. 1.
^q & Dat. 4.
^r || Dat. 40.
^s Def. 3.

Therefore, if a triangle be given in species, and from one of its angles a line be drawn perpendicular to its opposite side, that perpendicular has a given ratio to that side which it cuts; which was to be demonstrated.

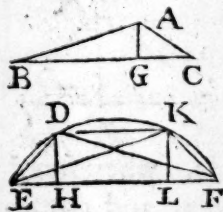
LEMMA

L E M M A.

This is Proposition 79th in the other copies.

If two triangles have one angle of the one equal to one angle of the other, and from these equal angles perpendiculars are drawn to the bases, and if the bases have the same proportion to their respective perpendiculars, the triangles are similar.

Let ABC , DEF be two triangles which have the angles BAC , EDF equal to one another, and from the angular points A and D let the lines AG , DH be drawn perpendicular to the Bases BC , EF , and let BC have the same proportion to AG that EF has



to DH ; I say the triangles ABC , DEF are similar. For, about the triangle DEF let a circle be described^f, as $EDKF$, with the line EF , and at the point E , construct the angle FEK equal to CBA ^g, from the point K draw KL perpendicular to EF ^h, join the lines KF , KD ; then, because the angle ABC is equal to KEF , and the angles AGB , KLE are right angles, whereforeⁱ the triangles ABG , KEL are similar; therefore as BG is to AG , so is LE to KL ; for the same reason as GC is to AG , so is FL to KL , and consequently, by compounding the antecedents, as BC is to AG , so is EF to KL ; but as BC is to AG , so is EF to

^f Prop. 5.
Euc. lib. 4.

^g Prop. 23.
Euc. lib. 1.

^h Prop. 12.
Euc. lib. 1.

ⁱ Prop. 32.
Euc. lib. 1.
& Prop. 4.
Euc. lib. 6.

to DH^x ; therefore as EF is to KL , so is EF^x Hypoth. to DH , consequently $^y DH$ is equal to KL ; ^{y} Prop. 9. but DH is parallel to KL , both being perpendicular to EF , wherefore $^z DK$ is parallel and ^{z} Prop. 33. equal to HL . Again, because DK is parallel ^{a} Prop. 29. to HL^x , the angle DKE is equal to the angle KEF ; but KEF is equal to the angle ABC , ^{b} Prop. 21. and the angle DKE is equal to DFE^b ; wherefore the angle DFE is equal to the angle ABC , ^{c} Prop. 32. and consequently ^{c} the remaining angle ACD is equal to the angle FED , and the triangle ABC is similar to the triangle DEF^d . ^{d} Prop. 4.

Therefore, if two triangles have one angle of the one, equal to one angle of the other, and from these equal angles perpendiculars are drawn to the bases, and if the bases have the same proportion to their respective perpendiculars, the triangles are similar; which was to be demonstrated.

This lemma is a remarkable instance of the great corruption of the manuscript copies. For, after Pappus mentions the last seven propositions concerning parallelograms, he says the next that follow are six, two of which are concerning triangles, and the other four treat of the properties of more than four proportional lines. It is certain that the last proposition cannot be one of the corollaries he means, which are laid down as consequences to the preceding seven propositions, because it has no manner of connection with them; therefore it must be one of the two that he speaks of in this series,

or

or it has not been inserted in any of the copies seen by Pappus: in all the other copies there are three inserted in this series that treat of triangles, consequently one of them must be spurious. The above lemma, which in all the copies is used as one of them, is plainly no way concerning the doctrine of the Data, but an elementary proposition; and its proper place in Euclid's Elements, had it been inserted, would have been after the seventh proposition in the sixth book; and since it is not, if the demonstration of the following proposition be genuine, which intirely depends upon this lemma, Euclid, according to his own and the constant practice of the Antients, laid it down as a lemma for that purpose. Had it been otherwise, Pappus would have taken notice of it, and consequently, in place of two he would have told us there are three which treat of triangles, and in that case the series must have consisted of seven propositions, and not of six; upon the whole, it is absurd to insert this lemma as one of the propositions in the Data: neither does it appear probable, that it was inserted in any of the copies of the Data extant in Pappus's time. For, in all the other writings of the Antients upon resolution and composition, he carefully enumerates the lemmas as well as the theorems or problems; but takes not the least notice of this. And what considerably strengthens this opinion, is, that the second demonstration to the following proposition, which does not depend upon the lemma, is
much

much more elegant, accurate, and natural, than the first which does, and therefore, by far, has a greater probability of being the genuine demonstration used by Euclid. The first demonstration is by no means so natural, as it necessarily requires the introduction of this Lemma; neither is the conclusion free of ambiguity, because there is no step in the whole demonstration that proves the position of the line HK * to be such, as either to touch or cut the segment EDF, upon which the whole demonstration depends; but as the line EK may fall without the circumference EDF, at least there is no step in the demonstration to prove the contrary; therefore the first demonstration of the following proposition, which depends upon the above lemma, is inconclusive. Mr. Hardy was sensible of this, and endeavours to free it from that ambiguity, in a note to the proposition, which is, "The right line HK must either touch or
 " cut the circumference of the segment FKG;
 " if the sides about the given angle be equal,
 " the line HK will touch the circumference;
 " but if these lines be unequal, the line HK
 " will cut the circumference of the segment
 " FKG, and therefore in both these cases the
 " point K by the twenty fifth Data is given.

This is his whole explanation, which appears to us to be no more than taking the very step in question for granted.

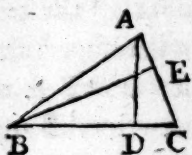
* See the figure to the following proposition.

PROPOSITION LXXV.

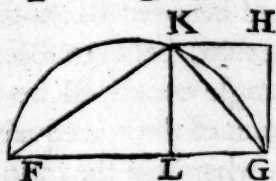
If a triangle has one angle given, and if the rectangle contained between the two sides about the given angle has a given ratio to the square of the third side, the triangle is given in species.

because the angle BAC is given^e, and the angle • Hypoth.
BEA is a right angle, wherefore the remaining
angle EBA^f is given, and consequently^g the tri-^f Prop. 32.
angle ABE is given in species; wherefore^h the Euc. lib. 1.
ratio of AB to BE is given. But, as AB is to^g Dat. 4.
BE, so is the rectangle contained between AB and^h Def. 3.
AC to the rectangle contained between BE andⁱ Dat. 40.
ACⁱ; but the rectangle contained between BE^j Prop. 1.
and AC is equal to the rectangle contained be-^k Euc. lib. 6.
tween BC and AD, because each of them is equal
to double the triangle ABC^k; wherefore the ratio^l Prop. 41.
of the rectangle contained between AB and AC^m Euc. lib. 1.
to the rectangle contained between AD and BC
is given; but the ratio of the rectangle contained
between BA and AC to the square of BC is
given^l, wherefore^m the ratio of the rectangleⁿ Hypoth.
contained between AD and BC to the square of^o Dat. 8.
BC is given; butⁿ as the rectangle contained^p Prop. 7.
between AD and BC is to the square of BC, so^q Euc. lib. 6.
is AD to BC; wherefore the ratio of AD to
BC is given. Again, let FG be a right line
given in magnitude and position, and upon FG
describe a segment of a circle containing an
angle in it, equal to the given angle BAC^o,^r Prop. 33.
which let be FKG; at the point G draw a line^s Euc. lib. 3.
at right angles to FG, which let be GH^p; ^r Prop. 11.
find a fourth proportional to BC, AD, and FG,^t Euc. lib. 1.
which let be GH^q; through the point H draw^u Prop. 12.
HK parallel to FG^r, which cuts the circumfe-^v Euc. lib. 6.
rence of the circle in K; from K draw KL^w Prop. 31.
perpendicular to FG^t, and join the lines FK^x Euc. lib. 1.
and^y Prop. 12.
L^z Euc. lib. 1.

and KG; then, because BC is to AD as FG is to GH, and the ratio of BC to AD is given, where-



^t Dat. 2.



^v Dat. 29.

^w Dat. 27.

^x Dat. 28.

^y Def. 3.

Dat.

^z Dat. 25.

^a Dat. 26.

^b Dat. 39.

^c Prop. 34.

Euc. lib. 1.

^d Constr.

tion.

fore the ratio of FG to GH is given; but FG is given in magnitude, wherefore GH is given in magnitude^t; but because FG is given in position, and the angle FGH is a right angle, therefore GH^v is given in position; but it is given

in magnitude, and the point G is given, where-

fore^w the point H is given, wherefore HK^x is

given in position. Again, because FG is given

in magnitude and position, and the angle FKG

is given, being equal to BAC, wherefore the

segment FKG is given in position^y, conse-

quently^z the point of intersection K is given.

But the points F and G are each given, where-

fore^a the lines KF, KG are each given in mag-

nitude, and consequently^b the triangle KFG is

given in species. But because BC is to AD, as

FG is to GH, and GH is equal to KL^c, where-

fore as BC is to AD, so is FG to KL, and the

angle BAC is equal to the angle FKG^d, where-

fore, by the preceding lemma, the triangle

ABC is similar to the triangle FKG; but FKG

is given in species, wherefore the triangle ABC

is given in species.

Therefore, if a triangle has one angle given, and if the rectangle contained between the two sides about

about the given angle has a given ratio to the square of the third side, the triangle is given in species; which was to be demonstrated.

This proposition is also, in the other copies, demonstrated in the following manner.

Let BAC be a triangle having the angle BAC given, and the ratio of the rectangle contained between BA and AC

to the square of BC given;

I say the triangle ABC is

given in species. For, since

the angle BAC is given^e,

the ratio of the excess by

which the square of BA and

AC, considered as one line, exceeds the square

of the third side BC to the triangle ABC is

given: let the figure DEF be equal to that

excess. But^f the ratio of the rectangle con-^f Dat. 65.

tained between BA and AC to the triangle ABC

is given, wherefore^g the ratio of the rectangle^g Dat. 8.

contained between BA and AC to the figure

DEF is given; but^h the ratio of the rectangle^h Hypoth.

contained between BA and AC to the square of

BC is given, thereforeⁱ the ratio of the figureⁱ Dat. 8.

DEF to the square of BC is given; wherefore^k ^k Dat. 6.

the ratio of the figure DEF, together with the

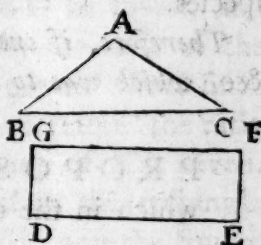
square of BC, to the square of BC is given.

But the figure DEF, together with the square

of BC, is equal to the square of AB and AC,

L. 2

con-



* Dat. 66.

considered as one line, therefore the ratio of the square of AB and AC, con-

sidered as one line, to the square of BC is given; wherefore¹ the ratio of BA and AC, considered as one line to BC, is given; but the angle BAC is given^m, consequentlyⁿ the triangle ABC is given in species.

Therefore, if one angle of a triangle be given, &c. which was to be demonstrated.

PROPOSITION LXXVI.

which in the others is Proposition 81.

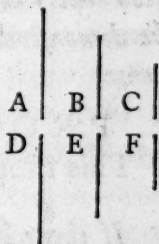
If there be three right lines in continued proportion, and other three likewise in continued proportion, and if the ratios of the extremes to each other be given, the ratio of the means to one another is also given; and also, if the ratio of the means, and likewise that of two of the extremes, be given, the ratio of the other two extremes will also be given.

Let A, B, C, be three right lines in continued proportion, and D, E, F, other three likewise in continued proportion, and let the ratios of the extremes, A to D, and C to F, be given; I say the ratio of the means, B to E, is also given. For since the ratios of A to D, and C to F are

• Dat. 69. each given, wherefore^o the ratio of the rectangle

gle

gle contained between A and C to the rectangle contained between D and F is given; but the rectangle contained between A and C is equal to the square of B^p, and the rectangle contained between D and F is equal to the square of E; wherefore the ratio of the square of B to the square of E is given, and consequently^q



^p Prop. 17.
Euc. lib. 6.

^q Dat. 54.

the ratio of the line B to the line E is given.

Again, let the ratio of A to D be given, and also the ratio of B to E; I say the ratio of the other extremes, C and F, is given. For, since the ratio of B to E is given, wherefore^r the ratio

^r Dat. 50.

of the square of B to the square of E is given.

But the square of B is equal to the rectangle

contained between A and C^f, and the square

^f Prop. 17.
Euc. lib. 6.

of E is equal to the rectangle contained between

D and F^t, therefore the ratio of the rectangle

^t Ibid.

contained between A and C to that contained

between D and F is given; and the ratio of A

to D is given^v. But two equiangular parallelo-

^v Hypoth.

grams having a given ratio to one another, and

one side of the one having a given ratio to one

side of the other^w, therefore the remaining side

^w Dat. 67.

C has a given ratio to the remaining side F.

Therefore, if there be three right lines in continued proportion, and other three likewise in continued proportion, and if the ratios of the extremes to each other be given, the ratio of the means to one another is also given; and also, if

the ratio of the means, and likewise that of two of the extremes, be given, the ratio of the other two extremes will also be given; which was to be demonstrated.

PROPOSITION LXXVII.

This Proposition is not in the other copies.

If there be four right lines in one order in proportion, and other four lines in another order likewise in proportion, and if the ratios of the extremes be given, each to each, and also the ratio of two of the means to one another, the ratio of the other two means to each other is also given; and, vice versa, if the ratio of the means to each other be given, and also the ratio of two of the extremes to one another, the ratio of the other extremes will also be given.

Let A, B, C, and D, be four right lines in one order in proportion, and E, F, G, and H,

A	B	C	D
E	F	G	H

four right lines in another order also in proportion, and let the ratios of the extremes to one another be given, each to each, that is the ratio of A to E, and of D to H; also let the ratio of one of the means to

one another be given, to wit, of B to F; I say the ratio of the other mean C to the other mean G is also given. For, since the ratios of A to

* Hypoth. E and D to H are given*, the ratio of the rectangle

angle contained between A and D to that contained between E and H is given^y. But because ⁷ Dat. 69. A is to B as C is to D^z, the rectangle contained ² Hypoth. between A and D is equal to the rectangle contained between C and B^a; for the same reason ^a Prop. 16. the rectangle contained between E and H is ^{Euc. lib. 6.} equal to the rectangle contained between F and G; wherefore the ratio of the rectangle contained between B and C to the rectangle contained between F and G is given; but the ratio of B to F is given^b. But two equiangular pa- ^b Hypoth. rallelograms having a given ratio to one another, and one side of the one having a given ratio to one side of the other^c, wherefore the ^c Dat. 67. remaining side of the one has a given ratio to the remaining side of the other, consequently the ratio of C to G is given. Again, let the ratios of B to F and C to G be given, and also the ratio of A to E; I say the ratio of D to H is given. For, since the ratio of B to F is given, and also the ratio of C to G^d, wherefore ^e the ^d Hypoth. ratio of the rectangle contained between B and ^e Dat. 67. C to that contained between F and G is given. But the rectangle contained between B and C is equal to the rectangle contained between A and D^f; also the rectangle contained between F and ^f Prop. 16. G is equal to that contained between E and ^{Euc. lib. 6.} H; wherefore the ratio of the rectangle contained between A and D to the rectangle contained between E and H is given; but the ratio of one side A to one side E is given; wherefore^g ^g Dat. 67.

the ratio of the other side D to the other side H is given*.

Therefore, if there be four right lines taken in one order in proportion, and other four lines in another order likewise in proportion, and if the ratios of the extremes be given, each to each, and also the ratio of two of the means to one another, the ratio of the other two means to each other is also given; and, vice versa, if the ratios of the means to each other be given, and also the ratio of two of the extremes to one another, the ratio of the other extremes will also be given; which was to be demonstrated.

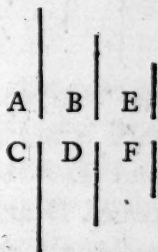
* Note, This proposition, by all probability, is the fourth proposition Pappus makes mention of concerning more than four lines in proportion, especially as it so immediately connects with the 76th proposition, and naturally ought to follow it. This corresponds to Euclid's method when he treats of proportional lines or numbers in the 6th and 7th books; the one he always makes to follow the other: it is therefore reasonable to presume that here he observed his constant method, and if he did, this which we have substituted must have been that which Pappus mentions, and which is wanting in all the other copies.

PROPOSITION LXXVIII.

This is Proposition 82 in other copies.

If there be four right lines in proportion, the first and third will have the same proportion to those lines which the second and fourth have the same given ratio to.

Let A, B, C, and D, be four lines in proportion, and let B have a given ratio to E, and as B is to E, so let D be to F; then will D have the same given ratio to F that B has to E; I say that A has the same proportion to E that C has to F. For, since A is to B, as C is to D^h, and B is to E, as D is to Fⁱ, wherefore^k, by equality, as A is to E, so is C to F.



^h Hypoth.
ⁱ Construc-
tion.
^k Prop. 22.
Euc. lib. 5.

Therefore, if there be four right lines in proportion, the first and third will have the same proportion to those lines, which the second and fourth have the same given ratio to; which was to be demonstrated.

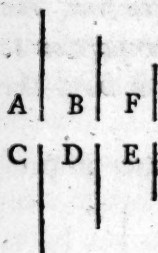
PROPOSITION LXXIX.

This is Proposition 83 in the other copies.

If there be four right lines, and to three of them, taken in any order, a fourth proportional be found in such a manner that the remaining fourth given line has a given ratio to it, then will the first have

have a given ratio to that line which is a fourth proportional to the fourth, third, and second.

Let A, B, C, and D, be four right lines, and let three of them be taken in any order, as



¹ Prop. 12.
Euc. lib. 6.

A, B, and C, to them let a fourth proportional be found, as E, in such a manner that the fourth line, D, has a given ratio to E, and let a fourth be found to D, C, and B¹, viz. the fourth, third, and second, which let be F; I say the first A

has a given ratio to F. For, since A is to B, as C is to E, the rectangle contained between A and E is equal to the rectangle contained be-

² Prop. 16.
Euc. lib. 6.

tween B and C^m; and as D is to E, so is the rectangle contained between A and D to the

³ Prop. 1.
Euc. lib. 6.

rectangle contained between A and Eⁿ; but the ratio of D to E is given, wherefore the ratio of the rectangle contained between A and E to that contained between A and D is given, consequently the ratio of the rectangle contained between A and D to that contained between B and C is given.

But, when two equiangular parallelograms have

• Dat. 56.
or 72.

a given ratio to one another^o, one side of the one has a given ratio to that right line which is a fourth proportional to the other side of that parallelogram, and the two sides of the other parallelogram; wherefore A has a given ratio to that line which is a fourth proportional to D, C, and B; but F is that fourth proportional, wherefore A has a given ratio to F.

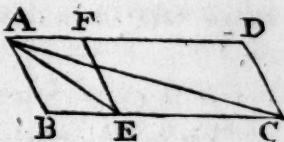
Therefore, if there be four right lines, and to three of them, taken in any order, a fourth proportional be found in such a manner that the remaining fourth given line has a given ratio to it, then will the first have a given ratio to that line which is a fourth proportional to the fourth, third, and second; which was to be demonstrated.

PROPOSITION LXXX.

which is Proposition 84 in other copies.

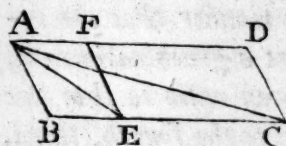
If a parallelogram, or a triangle, given in magnitude, be formed by two unequal right lines which contain a given angle, and if the excess, by which the greater exceeds the smaller, be given, each of these lines will be given in magnitude.

Let AB, BC, be two unequal lines which form the parallelogram BD, or the triangle ABC given in magnitude, and let the angle ABC be given, and also the excess, by which the greater, BC, exceeds the smaller, AB, be given; I say each of the lines, AB, BC, are given in magnitude. For, since BC exceeds AB by a line given in magnitude^p, let the excess be EC; take it ^p Hypoth. away, and the remainder, BE, is equal to AB; ^q through the point E draw EF, parallel to AB, ^q Def. 9. cutting AD in F; then, because AB is equal to BE, the ratio of AB to BE is given; but the angle ABE is given, wherefore the angle BAF



* Prop. 29.
 Euc. lib. 1.
 & Dat. 4.
 † Def. 3.
 Dat.
 ‡ Hypoth.

BAF is given[‡], consequently[†] the parallelogram BF is given in species. But BD is given in magnitude[‡], wherefore to the given right line EC the parallelogram BD, given in magnitude, is



applied, exceeding in figure by the parallelogram given in species; wherefore[‡] AB or BE is given in magnitude; but EC is given, wherefore[‡] BC is given. The same demonstration takes place, with respect to the triangle ABC, from the 41st of the first book, and 15th of the fifth book.

Therefore, if a parallelogram, or a triangle, given in magnitude, be formed by two unequal right lines which contain a given angle, and if the excess, by which the greater exceeds the smaller, be given, each of these lines will be given in magnitude; which was to be demonstrated.

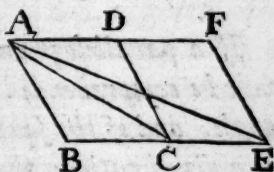
PROPOSITION LXXXI.

This is Proposition 85 in the other copies.

If two right lines comprehend a given parallelogram, or a triangle in a given angle, and if the sum of these lines be given, each of these lines are given in magnitude.

Let AB, BC be two lines which, in the given angle ABC, form the parallelogram BD, or the triangle ABC, given in magnitude, and let the sum of the two lines, AB, BC, be given; I say each

each of the lines, AB, BC, are given in magnitude. For, produce BC, and make CE equal to AB or CD, through the point E draw EF parallel to CD^x, and compleat the parallelogram BF^y; then, because DC is equal to CE, the ratio of DC to CE is given,



^x Prop. 31.
Euc. lib. 1.

^y Prop. 33.
Euc. lib. 1.

and the angle DCE is equal to ABC^z; but the angle ABC is given^a, wherefore the angle DCE is given, and consequently the remaining angle CDF^b is given, and therefore the parallelogram CF is given in species^c. Again, because the sum of AB and BC is given; but the sum of AB, BC is equal to BE, wherefore BE is given^d: hence to the given line BE there is applied the given parallelogram BD, wanting in figure by the parallelogram CF given in species; wherefore^e DC, CE are each given in magnitude; but DC is equal to AB, wherefore AB is given in magnitude; but BE is given, and CE is given, wherefore BC^f is given in magnitude. The same demonstration takes place with respect to the triangle ABC.

^z Prop. 29.
Euc. lib. 1.
^a Hypoth.

^b Prop. 29.
Euc. lib. 1.
& Dat. 4.
^c Def. 3.

^d Def. 10.
Dat.

^e Dat. 58.

^f Dat. 4.

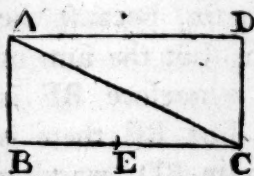
Therefore, if two right lines comprehend a given parallelogram, or a triangle in a given angle, and if the sum of these lines be given, each of these lines are given in magnitude; which was to be demonstrated.

PROPOSITION.

This is Proposition 86 in the other Copies.

If a parallelogram or triangle given in magnitude be comprehended by two right lines in a given angle, and if the square of the one line be more than in a given ratio to the square of the other by a given rectangle, each of these lines will be given in magnitude.

Let BD be a parallelogram given in magnitude, or ABC a given triangle, comprehended by the two lines AB and BC in the given angle ABC, and let the square of BC be more than in a given ratio to the square of AB by a given rectangle, which let be that contained between BC and BE; I say the lines AB, BC are each given in magnitude. For, the given angle ABC must either be a right angle or not. First let it be a right angle: then, because the square of BC is more than in a given ratio to the square of AB by the given



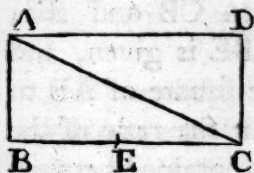
- ^z Hypoth. rectangle contained between CB and BE^z, take away that given rectangle, and the remainder, which is ^b the rectangle contained between BC and EC, will have a given ratio to the square of ABⁱ; and since the rectangle BD is given^k, and the rectangle contained between CB and BE is given, wherefore^l the ratio of the rectangle BD
- ^a Prop. 2. Euc. lib. 2.
ⁱ Def. 11.
^k Hypoth.
^l Dat. 1.

BD

BD to that contained between BC and BE is given; but^m as AB is to BE, so is the rectangle^m Prop. 1. BD to that contained between CB and BE; Euc. lib. 6. wherefore the ratio of AB to BE is given, and consequentlyⁿ the ratio of the square of AB toⁿ Dat. 50. the square of BE is given; but the ratio of the square of AB to the rectangle contained between BC and CE is given, wherefore^o the ratio of^o Dat. 8. the square of BE to the rectangle contained between BC and EC is given, and therefore the ratio of four times the rectangle contained between BC and CE to the square of BE is given, consequently^p the ratio of four times the^p Dat. 6. rectangle contained between BC and EC, together with the square of BE, to the square of BE is given. But, four times the rectangle contained between BC and EC, together with the square of BE, are equal to the square of BC and EC, considered as one line^q; wherefore the^q Prop. 8. ratio of the square of BC and EC, considered as Euc. lib. 2. one line, to the square of BE is given, and consequently^r the ratio of BC and EC, considered as^r Dat. 54. one line, to BE is given; wherefore^s the ratio^s Dat. 6. of BC, EC, and BE, added together, to BE is given, and consequently the ratio of BC to BE is given; but as BC is to BE, so is the rectangle contained between BC and BE to the square of BE^t; wherefore the ratio of the rectangle^t Dat. 1. contained between BC and BE to the square of BE is given; but the rectangle contained between BC and BE is given in magnitude, wherefore

¶ Dat. 2. fore^v the square of BE is given in magnitude, and consequently the line BE is given in magnitude; but it has been demonstrated that the^w ratio of BE to BA is given, and

¶ Dat. 55.



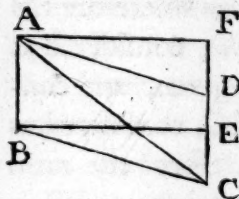
wherefore BA is given in magnitude. Hence upon the given line AB there is described the given parallelogram BD, containing a given angle ABC, therefore^x the other side, BC, is given in magnitude.

¶ Dat. 57.

The same demonstration takes place with respect to the triangle ABC, by completing the parallelogram BD in the same manner as has been observed in several antecedent propositions.

Again, let the given angle ABC be not a right angle, with the line AB, and at the point B, construct the angle ABE

¶ Prop. 11.
Euc. lib. 1.
¶ Prop. 31.
Euc. lib. 1.



a right angle^y, and complete the rectangle BF^z; then, because the parallelograms BD, BF are upon the same base, and between the same paral-

¶ Prop. 35.
Euc. lib. 1. els, therefore^a the parallelogram BD is equal to BF; but BD is given in magnitude, therefore BF is given in magnitude. Again, because

¶ Hypoth. the angle ABC is given^b, and the angle ABE is a right angle, wherefore^c the angle EBC is given; but the angle BEC is a right angle, wherefore the remaining angle BCE is given,

consequently^d the triangle BEC is given in^d Dat. 40. species; therefore the ratio of BE to BC is given^e, and wherefore^f the ratio of the square^e Def. 3. of BE to the square of BC is given; but the^{Dat.} square of BC is more than in a given ratio to^f Dat. 50. the square of AB by a given rectangle or space, wherefore^g the square of BE is more than in a^g Dat. 13. given ratio to, the square of AB by a given rectangle or space; and they contain the rectangle BF given in magnitude, consequently, by the above demonstration, AB and BE are both given in magnitude. Hence the given parallelogram BD is applied to the given line AB in a given angle, therefore^h the other side BC^h Dat. 57. is given in magnitude. The same demonstration takes place with respect to triangles.

Therefore, if a parallelogram or triangle given in magnitude be comprehended by two right lines in a given angle, and if the square of one of the lines be more than in a given ratio to the square of the other by a given rectangle, each of these lines will be given in magnitude; which was to be demonstrated.

PROPOSITION LXXXII.

This is Proposition 87 in the other copies.

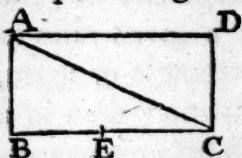
If a parallelogram or a triangle given in magnitude be comprehended by two lines in a given angle, and if the square of one of these lines exceed the square of the other by a given rectangle or

M

space,*

*space**, each of these lines will be given in magnitude.

Let AB, BC be two lines which comprehend the parallelogram BD, or the triangle ABC, given in magnitude, in the given angle ABC, and let the square of BC exceed the square of AB by a given space, to which let the rectangle contained be-



¹ Prop. 45.
Euc. lib. 1.

between BC and BE be equal¹; I say the two lines AB and BC are each given in magnitude. For the given angle ABC must either be a right angle or not. And first let it be a right angle: then, because the square of BC exceeds the square of AB by the rectangle contained be-

² Hypoth.

¹ Prop. 2.

Euc. lib. 2.

^m Def. 9.

Dat.

ⁿ Hypoth.

^o Dat. 1.

tween BC and BE^k, take away that rectangle, and the remainder, which¹ is the rectangle con-

tained between BC and EC, is^m equal to the square of AB. Again, since the rectangle BD

is givenⁿ, and the rectangle contained between

BC and BE is given, wherefore^o the ratio of the rectangle BD to that contained between BC

and BE is given; but the rectangle BD is to

that contained between BC and BE, as AB is

^p Prop. 1

Euc. lib. 6.

^q Dat. 50.

to BE^p, wherefore the ratio of AB to BE is

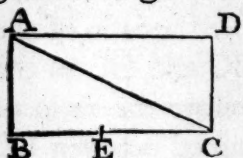
given, and consequently^q the ratio of the square

** In this and the preceding proposition the word space in the excess may be used; for whatever figure it is of, by Prop. 45, Euc. lib. 1. a rectangle may be made equal to it.*

of AB to the square of BE is given; but the square of AE is equal to the rectangle contained between BC and EC, wherefore the ratio of the rectangle contained between BC and EC to the square of BE is given; therefore the ratio of four times the rectangle contained between BC and EC to the square of BE is given; and by compounding them^r, the ratio of four times the ^r Dat. 6, rectangle contained between BC and EC, together with the square of BE to the square of BE is given. But, four times that rectangle contained between BC and EC, together with the square of BE, are equal to the square of BC and EC, considered as one line^t, wherefore the ratio ^t Prop. 8, of the square of BC and CE, considered as one ^t Euc. lib. 2. line, to the square of BE is given, and therefore^t ^t Dat. 54, the ratio of BC and EC, considered as one line, to the line BE is given; and by compounding them,^v ^v Dat. 6, the ratio of BC, together with EC and EB, that is double BC, to EB is given, wherefore the ratio of BC to BE is given. But, as BC is to BE, so is the rectangle contained between BC and BE to the square of BE^w, wherefore the ratio of the rectan- ^w Prop. 1, gle contained between BC and BE to the square of ^w Euc. lib. 6, BE is given; but the rectangle contained between BC and BE is given in magnitude, therefore^x the ^x Dat. 2, square of BE is given in magnitude, and consequently^y the line BE itself is given in magnitude; ^y Dat. 55, but it has been demonstrated that the ratio of BE to AB is given, and BE is given in magnitude, wherefore^z AB is given in magnitude. Hence to ^z Dat. 2,

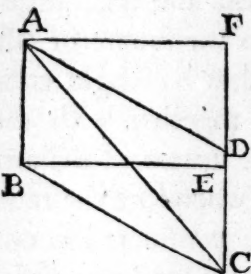
the line AB, given in magnitude, the parallelogram BD, given in magnitude, is applied in the

^a Dat. 57.



given angle ABC, wherefore ^a the other side, BC, is also given in magnitude. The same demonstration takes place with respect to the triangle ABC, as we have observed in the former proposition.

^b Prop. 11.
Euc. lib. 1.



Again, let the given angle ABC be not a right angle, with the line AB, and at the point B, draw BE at right angles to AB^b, and compleat the rectangle BF; then is the demonstration the same as in the second

part of the preceding proposition; and therefore, if a parallelogram or a triangle given in magnitude be comprehended by two lines in a given angle, and if the square of one of these lines exceed the square of the other by a given rectangle or space, each of these lines will be given in magnitude; which was to be demonstrated.

Note, In all the copies we have seen, this last series of propositions is extremely vitiated; because, in their enuntiations, the areas formed by the lines are always mentioned spaces in general; whereas, by the nature of them, it clearly appears, that they are limited to parallelogramick or triangular spaces in the same manner as in our note to propositions

positions 57, 58, and 59; otherwise none of them are demonstrated. It is true, Pappus makes mention of spaces when he names this series; but his meaning is applicable to no other than these two species of figures. The two last propositions are not demonstrated in any of the copies; for, in their hypothesis, they suppose the given angle of any magnitude, and, in the second step of the demonstration, they suppose it a right angle; and, as the whole demonstration depends upon that step, they have thereby deviated from the hypothesis which supposed it of any magnitude, by which the proposition is left undemonstrated.

So far they are from proving it from the hypothesis, which supposes the area comprehended by these lines may be any figure; that in their demonstration they have confined it to a rectangle alone; whereas we have demonstrated it to belong to any parallelogramick or triangular space; and to no other figure it can be applied. Dr. Gregory observes, that the Greek manuscript in the 86th proposition is vitiated in a very remarkable manner; nor did he see any way of recovering it, but by enuntiating it several different ways, and even then he solved it by an algebraick equation. By his enuntiation he has made it two distinct propositions. It is therefore highly probable, that from enuntiating it differently in different manuscripts, the fourth proposition has been introduced: for in all the copies we have four in this series, where Pappus mentions only three, which of the two is spurious is difficult, if not impossible, at this distance

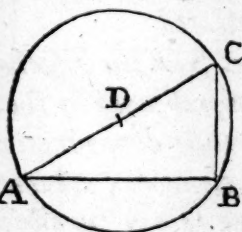
of time, to determine; for which reason we have inserted both: and that Pappus's account may be preserved, one only is numbered.

PROPOSITION LXXXIII.

This is Proposition 88 in other copies.

If within a circle, given in magnitude, a right line be drawn which cuts off a segment that contains in it a given angle, that right line is given in magnitude.

Let ABC be a circle given in magnitude, and AB a right line drawn within it which



cuts off the segment ACB, and let the angle in the segment ACB be given; I say the line AB is given in magnitude. For let D^c be the center of the circle, join

DA, which, produced, cuts the circumference at C, and join CB.

Then, because ACB is a semicircle, the angle ABC is a right angle^d, and the angle ACB is given^e, wherefore^f the remaining angle BAC is given, consequently^g the triangle ABC is given in species, and therefore^h the ratio of AC to AB is given; but AC is given in magnitude, wherefore ABⁱ is given in magnitude.

Therefore, if a right line be drawn within a circle given in magnitude which cuts off a segment that contains in it a given angle, that right line is given in magnitude; which was to be demonstrated.

P R O.

^c Prop. 1.
Euc. lib. 3.

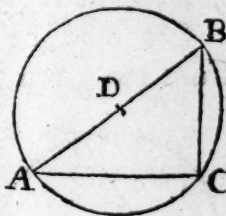
^d Prop. 31.
Euc. lib. 3.
^e Hypoth.
^f Prop. 32.
Euc. lib. 1.
& Dat. 4.
^g Dat. 40.
^h Def. 3.
Dat.
ⁱ Dat. 2.

PROPOSITION LXXXIV.

This is Proposition 89 in other copies.

If a right line given in magnitude be drawn within a circle given in magnitude, the angles contained in the segment cut off by that line are given in magnitude.

Let AC be a right line given in magnitude, drawn within the circle ABC which is also given in magnitude; I say the angles contained in the segment ABC are given. For let D be the center of the circle¹, join DA, which, produced, cuts the circumference at B, join BC.



¹ Prop. 1.
Euc. lib. 3.

Then, because the circle ABC is given in magnitude, the diameter AB is given in magnitude^m; but AC is givenⁿ, wherefore^o the ratio^m of AB to AC is given; but because ACB is a semicircle, the angle ACB is a right angle^p; hence in the right angled triangle ABC the ratio of the two sides AB and AC about the acute angle BAC is given; wherefore^q the triangle ACB is given in species, and consequently^r the angle ABC is given.

^m Def. 5.
^{Dat.}
ⁿ Hypoth.
^o Dat. 1.
^p Prop. 31.
Euc. lib. 3.
^q Dat. 43.
^r Def. 3.
Dat.

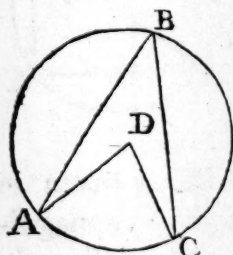
Therefore, if a right line given in magnitude be drawn within a circle given in magnitude, the angles contained in the segment cut off by that line are given in magnitude; which was to be demonstrated.

PROPOSITION LXXXV.

This is Proposition 90 in the other copies.

If from a given point in the circumference of a circle, given in position, a right line be drawn, cutting the circumference, and if from that point of intersection a line be drawn, making a given angle with the first mentioned line, the point where this last line, when produced, cuts the circumference of the circle is given.

Let ABC be a circle given in position, and from the given point A let AB be drawn, cutting the circumference at B, and from B let BC be drawn, so as to make the angle ABC given, let the line BC cut the circumference at C; I say the point C is given. For let D be the center of the cir-



cle^f, and join the lines DA and DC.
 Euc. lib. 3. Then, because the angle ADC is double the
 † Prop. 20. angle ABC^t, but the angle ABC is given^v,
 Euc. lib. 3. wherefore the angle ADC is given. Again,
 † Hypoth. because the circle ABC is given in position,
 † Def. 6. therefore its center D is given^w; but the point
 Dat. A is given, wherefore^x the line AD is given in
 † Dat. 26. position; but the angle ADC is given, there-
 † Dat. 29. fore the line DC^y is given in position; but the
 † Dat. 25. circle is given in position, wherefore^z the point
 of intersection, C, is given.

Therefore,

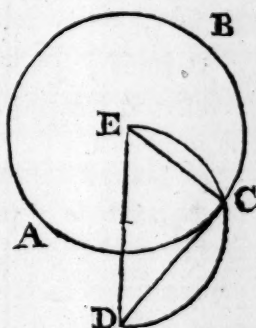
Therefore, if from a given point in the circumference of a circle, given in position, a right line be drawn, cutting the circumference, and if from that point of intersection a line be drawn, making a given angle with the first mentioned line, the point where this last line, when produced, cuts the circumference of the circle is given; which was to be demonstrated.

PROPOSITION LXXXVI.

This is Proposition 91 in the other copies.

If from a given point, without a circle given in position, a right line be drawn a tangent to the circle, that right line is given in position and magnitude.

Let ABC be a circle given in position, D a given point without it, and from the point D let DC be drawn a tangent to the circle ABC; I say DC is given in position and magnitude. For let E be the center of the circle ABC^a, and join the lines EC, ED, upon ED, as a diameter, describe the circle ECD. Then, because DC is a tangent, and CE is drawn to the center from the point of contact, wherefore^b the angle DCE is a right angle, therefore^c the circumference of the circle, which has ED for its diameter, must



^a Prop. 1.
Euc. lib. 3.

^b Prop. 18.
Euc. lib. 3.
^c Prop. 31.
Euc. lib. 3.

must pass through the point of contact, C. Again, because the circle ABC is given in position,

^d Def. 6.
Dat.

^e Dat. 26.

^f Def. 6.
Dat.

^g Dat. 25.

^h Dat. 26. given, consequently ^h the tangent DC is given in magnitude and position.

Therefore, if from a given point, without a circle given in position, a right line be drawn a tangent to the circle, that right line is given in position and magnitude; which was to be demonstrated.

PROPOSITION LXXXVII.

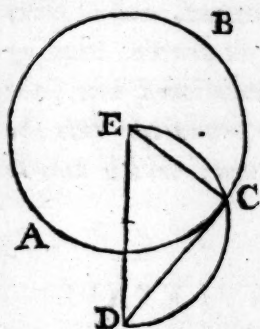
This is Proposition 92 in the other copies.

If from a given point, without a circle given in position, a right line be drawn, cutting the circle, and produced to the concavity, the rectangle contained between the whole secant line and that part without the circle, intercepted between the given point and the convexity, is given in magnitude.

Let ABC be a circle given in position, and from the given point D, without it, let the line

I

DB



DB be drawn, cutting the circle, and produced to the concavity at B; I say the rectangle contained between DB and DC

is given in magnitude. For from the point D draw DA a tangent to the circle ⁱ.

Then, because DA is a tangent to the circle ABC given in position, and drawn from

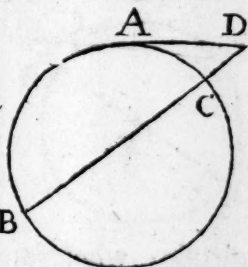
the given point D, wherefore ^k DA is given in ^k Dat. 87. magnitude and in position, consequently ^l the ^l Dat. 52.

square of DA is given in magnitude; but the square of DA is equal to the rectangle contained between DB and DC ^m, wherefore the rectangle ^m Prop. 36. contained between DB and DC is given in mag- Euc. lib. 3.
nitude.

Therefore, if from a given point, without a circle given in position, a right line be drawn, cutting the circle, and produced to the concavity, the rectangle contained between the whole secant line and that part without the circle, intercepted between the given point and the convexity, is given in magnitude; which was to be demonstrated.

This Proposition is also demonstrated in the following manner.

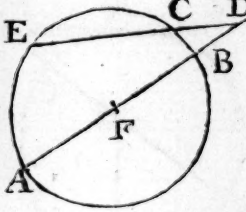
Find the center of the circle, which let be F ⁿ, ⁿ Prop. 1. join FD, which, produced, cuts the circumfe- Euc. lib. 3.
rence in A; then, because the circle ABC is given in position, ^o its center F is given; but ^o Def. 6.
the Dat.



ⁱ Prop. 17.
Euc. lib. 3.

^m Prop. 36.
Euc. lib. 3.

^p Dat. 26. the point D is given, wherefore ^p the line DF is given in position and magnitude; but the

^q Dat. 25.  circle ABC is given in position, therefore ^q the points of intersection, A and B, are each given, and consequently ^r the lines AD and BD are each given in mag-

^r Dat. 26. nitude; wherefore the rectangle contained between AD and BD is given in magnitude. But the rectangle contained between AD and BD is equal to the rectangle contained between ED and DC^f, wherefore the rectangle contained between ED and DC is given in magnitude.

^f Cor. to Prop. 37. Euc. lib. 3.

Therefore, if from a given point, &c. which was to be demonstrated.

PROPOSITION LXXXVIII.

This is Proposition 93 in the other copies.

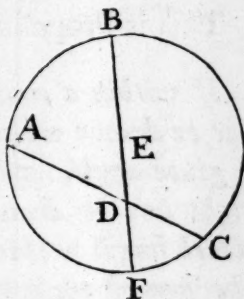
If from a given point, within a circle given in position, a right line be drawn, and produced both ways till it cut the circumference, the rectangle contained between the segments of that line, intercepted between the given point and the concavity of the circle, is given in magnitude.

Let ABC be a circle given in position, and D a given point within it, and from the point D let the line ADC be drawn, which, produced, cuts the circumference in A and C; I say the rectangle contained between AD and DC is

I

given

given in magnitude. For, find the center of the circle^t, which let be E, join DE, which, pro-^t Prop. 1. Eucl. lib. 3. duced, cuts the circumference in B and F; then, because the circle ABC is given in position, therefore^v its center E is given; but the point D is given^w, therefore^x the line ED is given in position and magnitude; but the circle is given in position, therefore^y the points of intersection, B and F,^y Dat. 25. are given, consequently^z the lines BD and DC^z Dat. 26. are given in magnitude, wherefore the rectangle contained between BD and DF is given in magnitude; but the rectangle contained between BD and DF is equal to the rectangle contained between AD and DC^a, wherefore the rectangle^a Prop. 35. Eucl. lib. 3. contained between AD and DC is given in magnitude.



^v Def. 6.
Dat.
^w Hypoth.
^x Dat. 26.

Therefore, if from a given point, within a circle given in position, a right line be drawn, and produced both ways till it cut the circumference, the rectangle contained between the segments of that line, intercepted between the given point and the concavity of the circle, is given in magnitude; which was to be demonstrated.

P R O-

PROPOSITION LXXXIX.

This is Proposition 94 in the other copies.

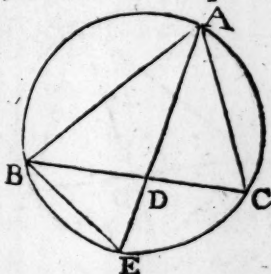
If within a circle given in magnitude a right line be drawn which cuts off a segment containing a given angle, and if from the angular point a right line be drawn, bisecting the given angle, which line is produced till it cut the circumference, the sum of these two lines, containing the given angle, has a given ratio to that line which bisects the angle; and the rectangle contained between these lines that contain the given angle, considered as one line, and that part of the line which bisects the angle, intercepted between the line that forms the base of the segment, and the circumference, is given in magnitude.

This Proposition is commonly demonstrated three different ways.

FIRST.

Let ABC be a circle given in magnitude, within which the line BC is drawn, cutting off the segment BAC, which contains a given angle, and from the angular point at A let the line AD be drawn, bisecting the angle BAC, and cutting the line BC at D, and the circumference at E; I say the ratio of BA and AC, considered as one line, to the line AE is given, and the rectangle contained between BA and AC, considered

sidered as one line, and DE, that part of the line which bisects the angle BAC, and is intercepted between BC and the circumference, is given in magnitude. For, join the line BE;



Then, because the circle BAC is given in magnitude, and the angles BAC, BAE are each given, wherefore^b the lines BC, BE are each given in^b Dat. 83. magnitude. Again, because the angle BAC is bisected with the right line AD, therefore^c as^c Prop. 3. CA is to AB, so is CD to DB; and, by alter-^c Euc. lib. 6. nation^d, as AC is to CD, so is AB to BD,^d Prop. 16. consequently^e, as AC is to CD, so is AC and^e Euc. lib. 5. AB, added together, to BC. Again, because^e Prop. 12. the angle BAE is equal to DAC^f, and the an-^f Hypoth. gle ACB equal to AEB^g, therefore the remain-^g Prop. 21. ing angle ADC is equal to the remaining angle^g Euc. lib. 3. ABE^h, and consequentlyⁱ, as AC is to CD, ^h Cor. 2. so is AE to EB; but as AC is to CD, so isⁱ Prop. 32. BA and AC, taken together, to BC; there-ⁱ Euc. lib. 1. fore, as BA and AC, taken together, is to BC, ⁱ Prop. 4. so is AE to EB; and, by alternation^k, as BA^k Euc. lib. 6. and AC, taken together, is to AE, so is BC to BE; but BC and BE are each given in mag-^k Prop. 16. nitude, and therefore the ratio of BC to BE,^k Euc. lib. 5. is given, consequently the ratio of BA and AC, taken together, to AE is given.^l Dat. 1.

Again, I say the rectangle contained between BA and AC, considered as one-line, and the line

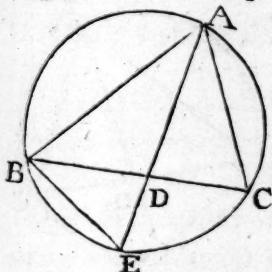
line DE is given in magnitude. Because the
^mProp. 21. angle ACD is equal to the angle DEB^m, and
 Euc. lib. 3.

ⁿProp. 15.
 Euc. lib. 1.

^oCor. 2.

Prop. 32.
 Euc. lib. 1.

^pProp. 4.
 Euc. lib. 6.



the angle ADC equal to BDEⁿ, therefore the angle DBE is equal to DAC^o, and consequently^p, as AC is to CD, so is BE to ED; but as AC is to CD, so is the sum of the lines BA and

AC to BC; wherefore as BA and AC, taken together, is to BC, so is BE to ED, and therefore^q the rectangle contained between BA and AC, taken together, considered as one line, and the line DE is equal to the rectangle contained between BC and BE. But the rectangle contained between BC and BE is given in magnitude, because each of the lines are given in magnitude, therefore the rectangle contained between BA and AC, taken together, considered as one line, and the line DE is given in magnitude.

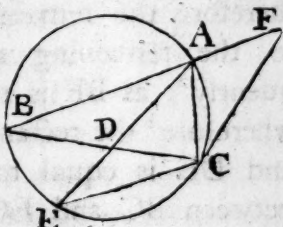
^qProp. 16.
 Euc. lib. 6.

Therefore, if within a circle given in magnitude a right line be drawn which cuts off a segment containing a given angle, and if from the angular point a right line be drawn, bisecting the given angle, which line is produced till it cut the circumference, the sum of these two lines, containing the given angle, has a given ratio to that line which bisects the angle; and the rectangle contained between these lines that contain the given angle, considered as one line, and that part of the line which

which bisects the angle, intercepted between the line that forms the base of the segment, and the circumference, is given in magnitude; which was to be demonstrated.

SECOND.

Produce the line BA, and make AF equal to AC, and join the lines EC and CF; then are the lines BC, CE each given in magnitude, as before, and consequently^r the ratio of BC to EC is given. Then, because AF is equal to AC, the



^r Dat. 1.

angles ACF, AFC are equal to one another^t,
and consequently^t each of them are equal to
half the given angle BAC. Hence the two
triangles, BCF, ACE, have the angle ABC
equal to the angle AEC^v, and the angle BFC
is proved equal to EAC, wherefore^w the re-
maining angle BCF is equal to the remaining
angle ACE; and therefore^x, as BF is to BC,
so is AE to EC; and, by alternation^y, as BF
is to AE, so is BC to EC; but the ratio of BC
to EC is given, wherefore the ratio of BF to
AE is given; but BF is equal to the sum of
the two lines, BA and AC, therefore the ratio
of the sum of the two lines, BA and AC, to
AE is given.

^t Prop. 5,

Euc. lib. 1,

^t Prop. 32,

Euc. lib. 1.

^v Prop. 21,

Euc. lib. 3,

^w Cor. 2.

Prop. 32,

Euc. lib. 1.

^x Prop. 4,

Euc. lib. 6,

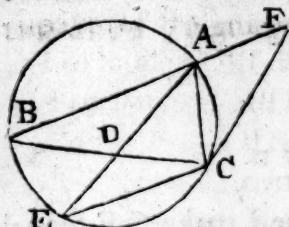
^y Prop. 16,

Euc. lib. 5.

N

Again,

Again, I say the rectangle contained between BA and AC, considered as one line, and DE



* Prop. 21.
Euc. lib. 3.

^a Prop. 4.
Euc. lib. 6.
^b Prop. 16.
Euc. lib. 6.

is given in magnitude, because the triangles BFC, DEC are equiangular. For the angle FBC is equal to AEC^a, and the angle BFC is equal to the angle DCE, each being equal to BAD, therefore the remaining angle BCF is equal to the remaining angle EDC, and consequently^a, as BF is to BC, so is EC to ED, wherefore^b the rectangle contained between BF and DE is equal to the rectangle contained between BC and EC; but the rectangle contained between BC and EC is given in magnitude, because each of the lines, BC and EC, are given in magnitude, wherefore the rectangle contained between BF, that is the sum of the two lines BA and AC, considered as one line, and the line DE is given in magnitude.

Therefore, if within a circle, &c. which was to be demonstrated.

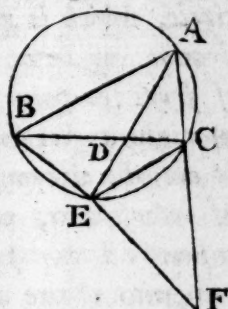
T H I R D.

Produce AC, and make CF equal to AB, and join the lines BE, EC, and EF.

Then, because the two angles ABE, ACE are equal to two right angles^c, and the two angles ACE, ECF are equal to two right angles^d, take away

^c Prop. 22.
Euc. lib. 3.
^d Prop. 13.
Euc. lib. 1.

away the common angle ACE; then will the remaining angle ABE be equal to the remaining angle ECF; and because the angle BAE is equal to EAC, therefore^e the arch BE is equal to EC, ^e Prop. 26. Euc. lib. 3. and consequently^f the line BE is equal to EC; ^f Prop. 29. Euc. lib. 3. then, because the two sides AB and BE are equal to the two sides FC and CE, each to each, and the angle ABE equal to FCE, therefore^g AE is equal to EF, the angle CFE equal to the angle BAE, and FEC equal to BEA. Again, because the two triangles BEC, AEF have the angles EAF, EBC equal^h, and the angle BEC equal to AEF, ^h Prop. 21. Euc. lib. 3. therefore the remaining angle BCE is equal to the remaining angle EFA; andⁱ as FA is to AE, so is BC to BE; but the ratio of BC to BE is given, because each of the lines BC, BE are given in magnitude^k, therefore the ratio of AF to AE is given; but AF is equal to the sum of the two lines BA and AC, wherefore the ratio of the sum of the two lines BA and AC, considered as one line, to AE is given. And, in the same manner as before, it is proved, that the rectangle contained between BA and AC, considered as one line, and the line DC is given in magnitude.



^g Prop. 4. Euc. lib. 1.

ⁱ Prop. 4. Euc. lib. 6.

^k Dat. 84.

Therefore, if within a circle, &c. which was to be demonstrated.

PROPOSITION XC.

This is Proposition 95 in the other copies.

If from any given point in the diameter of a circle, which is given in position, a right line be drawn any how, cutting the circumference, and if from the point of intersection, a line be drawn at right angles to it, which also is produced till it cut the circumference, and if from that point of intersection, where it cuts the circumference, a line be drawn parallel to the first mentioned line, the point where that last line cuts the diameter drawn through the given point, is given, and the rectangle contained between the two parallel lines, is given in magnitude.

Let ABC be a circle given in position, and D a given point in one of its diameters, as DB,



from D let DE be drawn, cutting the circumference at E, and from E, EC is drawn perpendicular to DE, and cutting the circumference at C, through C let FC be drawn parallel to DE, and cutting the diameter DB in F; I say the point F is given, and the rectangle contained between the parallel lines DE and CF is given in magnitude. For, let CF, produced, cut the circumference at G, and join GE.

Then,

Then, because DE is parallel to CF, and the angle DEC is a right angle, wherefore¹ ^{1 Prop. 29. Euc. lib. 1.} the angle ECG is a right angle, and consequently^m EG is the diameter of the circle^m ^{m Prop. 31. Euc. lib. 3.} ABC; but DF is a diameterⁿ, wherefore the pointⁿ ^{n Hypoth.} H, where these diameters cut each other, is the center of the circle ABC; but the center of the circle ABC is given^o, therefore H is given; ^{o Def. 6.} but D is given, therefore^p DH is given in ^{Dat.} magnitude and position. Again, because DE is parallel to CG, the angle DEH is equal to the angle HGF^q; and the angles EHD, FHG ^{q Prop. 29. Euc. lib. 1.} are equal^r, and the side EH is equal to HG, ^{r Prop. 15. Euc. lib. 1.} wherefore^f the triangle EDH is equal to the triangle HFG, and ED equal to FG, and DH ^{f Prop. 26. Euc. lib. 1.} to FH; but DH is given in magnitude and position, therefore HF is given in magnitude, and also in position; but the point H is given, therefore^t the point F is given. And because^t ^{t Dat. 27.} F is a given point, within the circle ABC, which is given in position, and from F the line CG is drawn, cutting the circumference of the circle, therefore^v the rectangle contained be- ^{v Dat. 88.} tween CF and FG is given in magnitude; but FG is proved equal to DE, wherefore the rectangle contained between the two parallel lines CF and DE is given in magnitude.

Therefore, if from any point given in the diameter of a circle, which is given in position, a right line be drawn any how, cutting the circumference, and if from the point of intersection, a line

line be drawn at right angles to it, which also is produced till it cut the circumference, and if from that point of intersection, where it cuts the circumference, a line be drawn parallel to the first mentioned line, the point where that last line cuts the diameter drawn through the given point, is given, and the rectangle contained between these two parallel lines, is given in magnitude; which was to be demonstrated.



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